Thermal Radiation, Heat and Mass Transfer Effects on MHD Fluid Flow over a Surface Embedded in a Non-Darcian Porous Medium

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Abstract:

A numerical simulation of unsteady MHD fluid flow over a surface embedded in a non-Darcian porous medium in presence of radiation, heat and mass transfer of a heat generation has been investigated. The governing equations of the problem contain a system of non-linear partial differential equations as well as non-similar are solved numerically by using the finite difference technique. Numerical results are presented to show the effects of various physical parameters on the velocity, temperature and concentration within the boundary layer. Finally, a qualitative comparison with previous published work is shown.

Keywords: Non-Darcian porous medium, Heat and mass transfer; Thermal radiation, Finite difference method.

1. INTRODUCTION

The effect of thermal radiation, heat and mass transfer on MHD boundary layer flow has become important in several industrial, scientific and engineering fields. The combined heat and mass transfer has great importance in the liquid metal flows, ionized gas flow in a nuclear reactor and for the application to the geophysics, aeronautics, astrophysics and Bio-engineering. The heat generation and thermal radiation with or without magnetic field suction or injection has been analyzed by several characters. There are many researchers have been presented engineering aspect of such flows with numerical solution of such problems of flow. The MHD flow of a rotating fluid past a vertical porous flat plate in the presence of chemical reaction and radiation has been investigated by Sivaiah [1]. Dufour and Soret Effects On Steady MHD Free Convection And Mass Transfer Fluid Flow Through A Porous Medium in A Rotating System have been investigated by Nazmul and Alam [2]. Bestman and Adjepong [3] investigated the unsteady hydro-magnetic free convection flow with radiative heat transfer in rotating fluid. Aurangziab et al. [4] studied the effects of Soret and Dufour on unsteady MHD flow by mixed convection over a vertical surface in porous media with internal heat generation, chemical reaction and Hall current.

Finite element analysis of hydro-magnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-Darcian porous medium in the presence of chemical reaction have been studied by Mohamed et al. [5]. Chauhan and Rastogi [7] studied Heat transfer effects on rotating MHD Couette flow in a channel with Hall current. The main objective of the present study is to the MHD Fluid Flow over a Vertical Porous Plate with Surface Embedded in a Non-Darcian Porous Medium in the presence of thermal radiation, mass transfer effect with Heat generation. Also our principle aim of this paper is to extend the work of Mohamed et al. [5] for the case where we consider the fluid in presence of thermal radiation and Heat generation effect Flow in a Non-Darcian Porous Medium. Lastly, we have compared the present results with the results of Mohamed et al. [5] has been shown in tabular form.
2. MATHEMATICAL FORMULATION

Let us consider an unsteady MHD mixed convective heat and mass transfer flow of an two dimensional incompressible, electrically conducting viscous fluid past an electrically non-conducting isothermal vertical porous plate with surface embedded in a non-darcian porous medium in the presence of thermal radiation and heat generation. The positive x coordinate is measured along the plate in the direction of fluid motion and the positive y coordinate is measured normal to the plate. A uniform transverse magnetic field of magnitude \( B_0 \) is applied in the direction of \( \text{y-axis} \). Initially, it is considered that the plate as well as the fluid is at the same temperature \( T(=T_\infty) \) and concentration level \( C(=C_\infty) \). Also it is assumed that the temperature of the plate and concentration are raised to \( T_w(>T_\infty) \) and \( C_w(>C_\infty) \) respectively, which are thereafter maintained constant, where \( T_w, C_w \) are temperature and concentration at the wall and \( T_\infty, C_\infty \) are the temperature and concentration of the species outside the boundary layer respectively. The physical configuration of the problem is furnished in Fig. 1. Within the framework of the above-stated assumptions the generalized equations relevant to the unsteady problem are governed by the following system of coupled partial differential equations as;

The Continuity equation;

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

The Momentum equation in \( x\)-axis ;

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0 u}{\rho} - \nu k u - C_i u^2 \tag{2}
\]

The Energy equation;

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p} (T - T_\infty) \tag{3}
\]

The Concentration equation;

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \tag{4}
\]

and corresponding boundary conditions are;

\[
u = U_0, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0
\]

\[
u = 0, \quad v = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty
\]
where \(u\), and \(v\) are the \(x\), and \(y\) components of velocity vector, \(\sigma\) is the electric conductivity, \(\nu\) are the kinematic coefficient viscosity, \(\rho\) is the density of the fluid, \(\kappa\) is the thermal conductivity, \(c_p\) is the specific heat at the constant pressure, \(D_m\) is the coefficient of mass diffusivity, \(k_i\) is the thermal diffusion ratio, \(T_m\) is the mean fluid temperature, \(Q\) is the heat absorption coefficient. The radiative heat flux \(\dot{q}_r\) is described by the Rossel and approximation (Brewster [6]) such that, \(\dot{q}_r = -\left(4\sigma^* / 3k^*\right) \left(\frac{\partial T^4}{\partial y}\right)\) where \(\sigma^*\) and \(k^*\) are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. This result in the following approximation: \(T^4 \approx 4T_m^4T - 3T_m^4\).

Let us introduced non-dimensional quantities are; \(X = xU_0/\nu\), \(Y = yU_0/\nu\), \(U = u/U_0\), \(V = v/U_0\), \(\nabla \cdot \mathbf{J} = 0\) for the current density \(\mathbf{J} = (J_x, J_y, J_z)\) where \(J_z\) is constant. Since the plate is non-conducting, \(J_y = 0\) at the plate and hence zero everywhere. \(\tau = tU_0^2/\nu\), \(\bar{T} = (T - T_m)/(T_m - T_\infty)\) and \(\bar{C} = (C - C_m)/(C_m - C_\infty)\). Substituting the above dimensionless variables in equations (1)-(4) and corresponding boundary conditions (5), the obtained dimensionless coupled non-linear partial differential equations are;

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{6}
\]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - (M + K)U - \Gamma U^2 \tag{7}
\]

\[
\frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} = \left(\frac{1 - R}{P_r}\right) \frac{\partial^2 \bar{T}}{\partial Y^2} + \beta \bar{T} \tag{8}
\]

\[
\frac{\partial \bar{C}}{\partial \tau} + U \frac{\partial \bar{C}}{\partial X} + V \frac{\partial \bar{C}}{\partial Y} = \frac{1}{S_c} \frac{\partial^2 \bar{C}}{\partial Y^2} \tag{9}
\]

and boundary conditions are;

\(U = 1, V = 0, \bar{T} = 1, \bar{C} = 1\) at \(Y = 0\)

\(U = 0, V = 0, \bar{T} = 0, \bar{C} = 0\) as \(Y \to \infty\) \tag{10}

Where \(\tau\) represents the dimensionless time, \(U\) and \(V\) are the \(X\) and \(Y\) components of dimensionless velocity vector, \(X\) and \(Y\) are the dimensionless Cartesian coordinates, \(\bar{T}\) is the dimensionless temperature, \(\bar{C}\) is the dimensionless concentration, and the parameters are given such as \(\Gamma = (C_\infty/U_0)\) (dimensionless porous medium inertia coefficient), \(K = \mu\nu/\rho\kappa'U_0^2\) (Permeability of the porous medium), \(M = \left(\sigma B_0^2\nu/\rho U_0^2\right)\) (Magnetic Parameter), \(R = (16\sigma^* T_m^4 / 3k^* \kappa)\) (Radiation parameter) \(P_r = (\rho c_p \nu/\kappa)\) (Prandtl Number), \(\beta = (Q_0/\rho c_p U_0^2)\) (Heat generation/Absorption parameter) and \(S_c = (\nu/D)\) (Schmidt Number)

3. NUMERICAL SOLUTIONS

The system of non-dimensional, nonlinear, coupled partial differential equations (6)-(9) with boundary condition (10) are solved numerically using explicit finite difference method.
To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes, where X-axis is taken along the plate and Y-axis is normal to the plate. Here the plate of height \( X_{\text{max}} = 100 \) is considered i.e. \( X \) varies from 0 to 100 and assumed \( Y_{\text{max}} = 30 \) as corresponding to \( Y \to \infty \) i.e. \( Y \) varies from 0 to 30. There are \( m(=100) \) and \( n(=100) \) grid spacing in the X and Y directions respectively as shown in Figure 2. It is assumed that \( \Delta X \), \( \Delta Y \) are content mesh size along X and Y directions respectively and taken as follows, 
\[ \Delta X = 0.50(0 \leq X \leq 100) \] and 
\[ \Delta Y = 0.15(0 \leq Y \leq 30) \] with the smaller time-step, \( \Delta \tau = 0.005 \). Let \( U', \bar{T}' \) and \( \bar{C}' \) denote the values of \( U \), \( T \) and \( C \) at the end of a time-step respectively. Using the explicit finite difference approximation, the following appropriate set of finite difference equations are obtained as;

\[
\frac{U'_{i,j} - U'_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \quad (11)
\]

\[
\frac{U'_{i,j} - U'_{i-1,j}}{\Delta \tau} + \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + (M + K)U_{i,j} - \Gamma U^2_{i,j} \quad (12)
\]

\[
\frac{\bar{T}'_{i,j} - \bar{T}'_{i-1,j}}{\Delta \tau} + \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} + \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j-1}}{\Delta Y} = \frac{(1 - R)}{Pr} \left( \bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1} \right) + \beta \bar{T}_{i,j} \quad (13)
\]

\[
\frac{\bar{C}'_{i,j} - \bar{C}'_{i-1,j}}{\Delta \tau} + \frac{\bar{C}_{i,j} - \bar{C}_{i-1,j}}{\Delta X} + \frac{\bar{C}_{i,j+1} - \bar{C}_{i,j-1}}{\Delta Y} = \frac{1}{Sc} \left( \bar{C}_{i,j+1} - 2\bar{C}_{i,j} + \bar{C}_{i,j-1} \right) \quad (14)
\]

with the boundary condition;

\[
U_{i,0}^n = 1, \quad V_{i,0}^n = 0, \quad \bar{T}_{i,0}^n = 1, \quad \bar{C}_{i,0}^n = 1 \quad (15)
\]

\[
U_{i,L}^n = 0, \quad V_{i,L}^n = 0, \quad \bar{T}_{i,L}^n = 0, \quad \bar{C}_{i,L}^n = 0 \text{ where } L \to \infty
\]

Here the subscript \( i \) and \( j \) designates the grid points with \( X \) and \( Y \) coordinates respectively and the superscript \( n \) represents a value of time, \( \tau = n\Delta \tau \) where \( n = 0, 1, 2, \ldots \ldots \). The velocity \( (U) \), temperature \( (\bar{T}) \) and concentration \( (\bar{C}) \) distributions at all interior nodal points have been computed by successive applications of the above finite difference equations.

4. RESULTS AND DISCUSSION

For investigating the physical situation of the problem, the numerical values and graphs of velocities \( (U) \), temperature \( (\bar{T}) \) and concentration \( (\bar{C}) \) distributions within the boundary layer have been computed for different values of Magnetic parameter \( (M) \), Permeability of the porous medium \( (K) \), Dimensionless porous coefficient \( (\Gamma) \), Heat generation Parameter \( (\beta) \), Radiation parameter \( (R) \), Prandtl number \( (Pr) \) and Schmidt number \( (Sc) \) with the help of a computer programming language Compaq Visual Fortran 6.6a and Tecplot 7. To obtain the steady-state solutions, the computation has been carried out up to \( \tau = 80 \) shown graphically. It is observed that the numerical values of \( U \), \( \bar{T} \) and \( \bar{C} \) however, show little changes after \( \tau = 40 \). Hence at \( \tau = 40 \) for all variables are steady-state solutions. The most important fluids are atmospheric air, water and methanol,
so that the results are limited to $P_r = 0.71$ (Prandtl number for air at $20^\circ C$), $P_r = 1.00$ (Prandtl number for salt water at $20^\circ C$) and $P_r = 7.00$ (Prandtl number for water at $20^\circ C$) and $P_r = 11.62$ (Prandtl number for methanol at $20^\circ C$).

To investigate the physical situation of the problem, the solutions have been illustrated in Figs. 3-10. The velocity and temperature profiles have been displayed for various values of Magnetic parameter $(M)$ in Figs. 3-4 respectively. These results show that the velocity profile decreases with the increase of Magnetic parameter $(M)$ and the reverse effect is found in the temperature profiles. In Fig.5, shown that the velocity profile slightly decreases with the increase of Dimensionless porous coefficient $(\Gamma)$. The Fluid concentration profiles have been illustrated for several values of heat generation or absorption parameter $(\beta)$ in Fig. 6. These results show that the fluid concentration profiles decreases with increase of heat generation parameter.

Fig. 3. Velocity profile for different values of Magnetic parameter with $\Gamma = 0.20, K = 1.00$, $R = 0.30, \beta = 0.50, P_r = 0.71, S_C = 0.60$

Fig. 4. Temperature profile for different values of Magnetic parameter with $\Gamma = 0.20, K = 1.00$, $R = 0.30, \beta = 0.50, P_r = 0.71, S_C = 0.60$

Fig. 5. Velocity profile for different values of dimensionless porous coefficient with $M = 2.00, K = 1.00$, $R = 0.30, \beta = 0.10, P_r = 0.71, S_C = 0.60$

Fig. 6. Concentration profile for different values of Heat generation parameter with $M = 2.00, K = 1.00$, $\Gamma = 0.20, P_r = 0.71, R = 0.30, S_C = 0.60$
The temperature profiles and velocity profiles have been displayed for various values of Radiation parameter ($R$) Permeability parameter ($K$) in Fig. 7 and Fig. 8 respectively. It is noted that the temperature profiles as well as velocity profiles decreases with the increases Radiation and Permeability parameter. The temperature profiles and fluid concentration profiles have been displayed for various values of Prandtl number ($Pr$) and Schmidt number ($Sc$) in Fig. 9 and Fig. 10 respectively. It is seen in the above mentioned figures that the temperature profiles as well as fluid concentration profiles decreases with the increases Prandtl number ($Pr$) and Schmidt number ($Sc$).
Moreover for the accuracy of the present results a qualitative comparison with the published results of Mohamed et al. [5] presented in Table 1.

Table-1: Comparison of the prediction of primary as well as velocity profiles, temperature profiles and concentration profiles for several physical parameters.

<table>
<thead>
<tr>
<th>Increased Parameter</th>
<th>Mohamed et al. [5]</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'$</td>
<td>Dec.</td>
<td>Inc.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Inc.</td>
<td>Dec. Inc.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Dec. Inc.</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>Dec.</td>
<td></td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this study, the explicit finite difference solution of MHD fluid flow over a Vertical Porous Plate with Surface Embedded in a Non-Darcian Porous Medium in the presence thermal radiation, heat transfer as well as heat generation is investigated. The physical properties are discussed for different values of parameters and important findings of this investigation are given below;

- The velocity profiles decreases with the increase of $K$, $M$, $\Gamma$.
- The temperature distributions decreases with the increase of $R$, $Pr$, and increase with the increase of $M$.
- The concentration profiles decreases with the increase of $\beta$, $Sc$.

REFERENCES