TORSIONAL CAPACITY ASSESSMENT OF COMPOSITE CROSS SECTIONS USING MATHEMATICA

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ABSTRACT
Torsional capacity is the ability of the cross section to resist a torque that attempts to twist the cross section. Mechanical properties, like shear stress and torsional stress, can be improved by using composite section rather than using single material. The main purpose of this paper is to improve the torsional capacity of Aluminium by combining with brass. Torsional capacity and shearing stress were calculated for five cross sections (Cases I, II, III, IV, and V), two of them were composed of one material (CS-I and II, Aluminium and Brass) and other three were composite sections with different arrangement of Aluminium and Brass (Case III and IV). Finite difference method was adopted to calculate the stress function, shearing stress and torsional capacity. The analysis of cross sections was performed in Wolfam Mathematica 8.0 software. The analysis result revealed that brass, individually, can provide higher torsional capacity among the studied materials and composite section having aluminium inside and brass outside, can bear highest shearing stress among the studied composite sections.

Keywords: Torsional stress; shear stress; composite section

INTRODUCTION
A composite material is a combination of two or more different materials. Composite section produces a new material with new properties, which are not found in the individual materials and it is the key advantage of using combination of two or more different materials. It has high performance due to the high specific strength, high specific stiffness, high flexibility, low density, low thermal expansion, and easy to fabricate. There are two phases of composite materials. The first one is reinforcing phase. In this phase, material exhibits high strength with low specific densities. The second phase is the matrix. In this phase, the material shows ductility and toughness, these properties are improved by other reinforcing materials. Composite materials can be generally involved into three categories, which are dispersion strengthened, particle reinforced, and fiber reinforced. In Dispersion strengthened, the matrix of the material is reinforced by fine distribution of secondary particles. The secondary particles affect the performance of the material causing deformation in it. Particle reinforced materials have many particles impeded in the matrix. In case of Fiber reinforced materials, materials are reinforced by fibers and most of the load carried by the fibers.
Torsional capacity is defined as the ability of the cross section to resist a torque that attempts to twist a cross section of its axis. In order to improve the torsional capacity for two or more materials, a combination of materials should be performed. Some researchers studied various cross section to assess the torsional and shear stress capacity (Massa and Barbero, 1998; Rao, 2007; Roberts and Al-Ubaidi, 2012; Salim and Davalos, 2005; Tarn and Wang, 2001).
The main purpose of this study is to improve the torsional capacity of Aluminium material combining with brass material and to find suitable arrangement of materials so that cross section can bear higher torsion and shearing stress.

METHODOLOGY
To investigate torsional capacity and shearing stress, square cross sections were chosen. Torsional capacity and shearing stress were found out for modulus of rigidity of two different materials by taking other parameters like dimensions and angle of twist as constant. To find out torsional capacity and shearing stress, the Saint Venant's theory of torsion and Prandtl stress function were used. Finite

difference method was used as analysis technique. All sections were analyzed to find out torsional capacity and shear stress by using Wolfam Mathematica 8.0 software.

The Prandlt Stress Function

The Poisson's equation can be written like Eq. (1).Where Φ is the Prandlt stress function and is angle of twist.

$$\nabla^2\Phi = -2G\text{Ø}' \qquad (1)$$

This equation can be rewritten as,

$$\nabla^2\bar{\Phi} = -2 \text{ in the cross section, and}$$
$$\bar{\Phi} = 0 \text{ on the boundary}$$

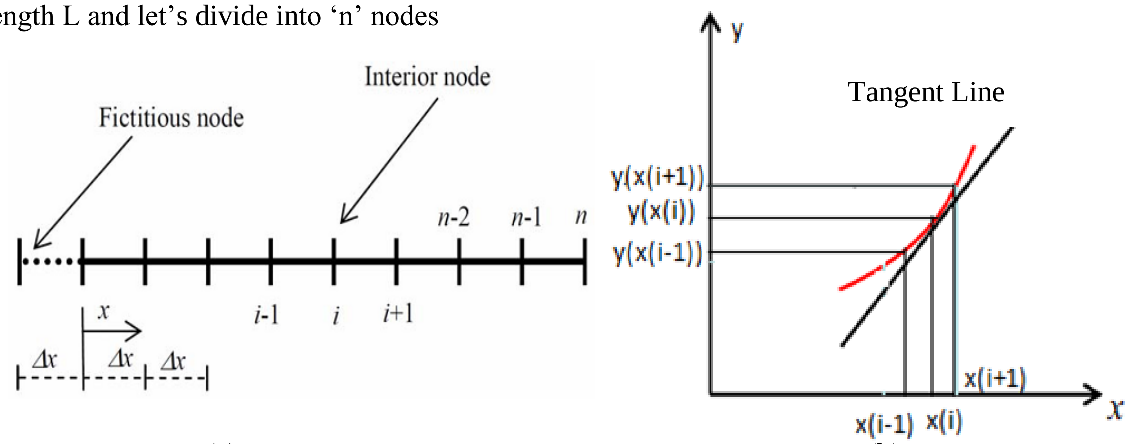
By using these governing equation and condition, Torque and shear stress can be determined as follows,

$$T = 2\int_A \Phi dA$$

$$\frac{\partial\Phi}{\partial n} = -\tau$$

Finite Difference Method

Finite difference method is a techniques of replacing ordinary differential equation, partial differential equation and their boundary conditions by a set of algebraic equations. Let's assume a 1-D structure with length L and let's divide into 'n' nodes



(a) (b)

Fig. 1: (a) Nodes taking for 1 D, (b) Graphical representation of nodes

So, the differential equation for all nodes can be written as

$$\frac{d^n y}{dx^n} + \frac{d^{n-1}y}{dx^{n-1}} + \dots\dots\dots\dots = f(x)$$

Now approximating each differential operator by a finite difference. From Taylor series, for first and second order central finite difference can be written as

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

For 2-D, second order central finite different can be written as, (in y and z direction)

$$y''_i = \frac{y_{i-1,j} - 2y_{i,j} + y_{i+1,j}}{{h_y}^2}$$

$$y''_j = \frac{y_{i,j-1} - 2y_{i,j} + y_{i,j+1}}{{h_z}^2}$$

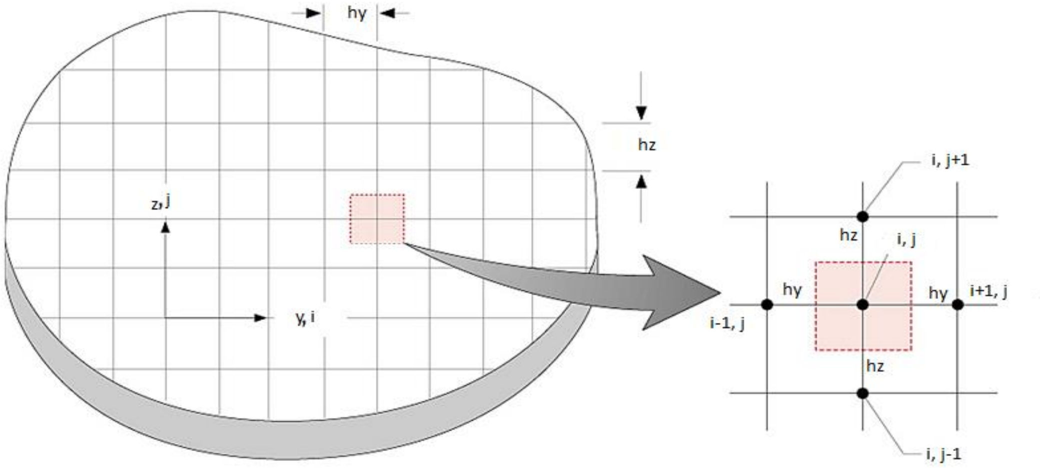


Fig. 2: Nodes taking for 2-D

From the derivation of stress function,

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = -2G\phi'$$

Here,

$$\frac{\partial^2\Phi}{\partial y^2} = \frac{\Phi_{i-1,j} - 2\Phi_{i,j} + \Phi_{i+1,j}}{{h_y}^2}$$

$$\frac{\partial^2\Phi}{\partial z^2} = \frac{\Phi_{i,j-1} - 2\Phi_{i,j} + \Phi_{i,j+1}}{{h_z}^2}$$

So,

$$\nabla^2\Phi = \frac{\Phi_{i-1,j} - 2\Phi_{i,j} + \Phi_{i+1,j}}{{h_y}^2} + \frac{\Phi_{i,j-1} - 2\Phi_{i,j} + \Phi_{i,j+1}}{{h_z}^2}$$

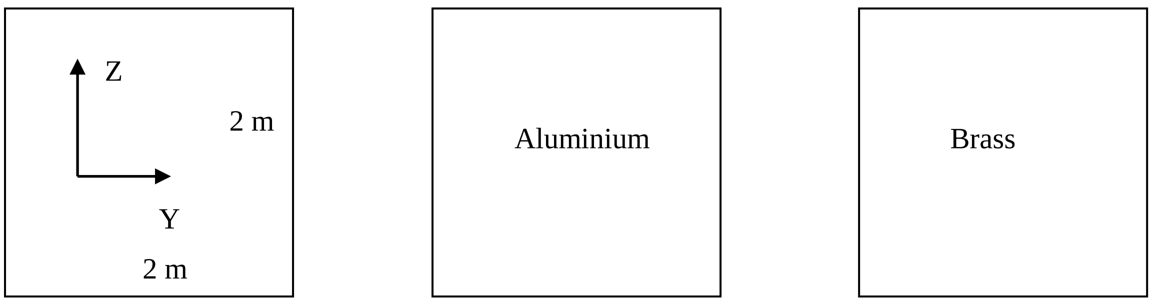
For $h_x = h_y = h$

$$\nabla^2\Phi = \frac{\Phi_{i-1,j} + \Phi_{i+1,j} - 4\Phi_{i,j} + \Phi_{i,j-1} + \Phi_{i,j+1}}{h^2}$$

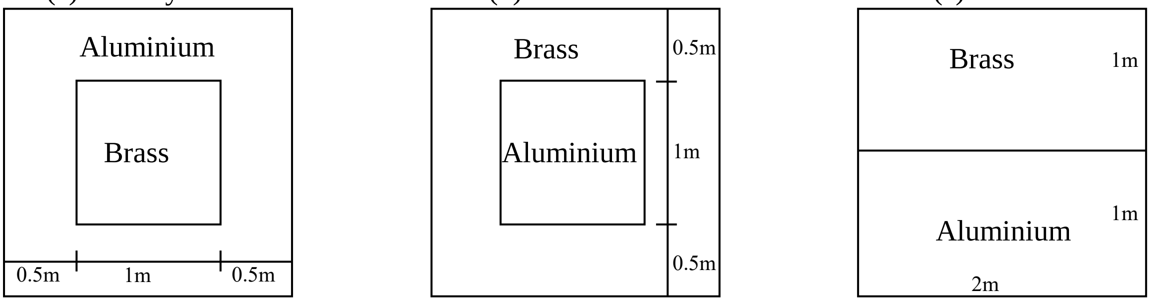
Experimental Cross Sections

To investigate torsional capacity and shearing stress square cross sections were chosen. Torsional capacity and shearing stress was found out for different modulus of rigidity by taking other parameters like dimensions and angle of twist as constant. Modulus of rigidity was taken as variable for different cross sections [Fig. 3].

The parameters are as follows,
Length in Y direction = 2 meter
Length in Z direction = 2 meter
Angle of twist, θ = 1 radians
Modulus of rigidity of Aluminium = 24000 MPa.
Modulus of rigidity of Brass = 40000 MPa.



(a) Axis System (b) CS-I (c) CS-II



(d) CS-III (e) CS-IV (f) CS-V

Fig. 3: Different Cross Sections

RESULTS AND DISCUSSIONS

Result was recorded for four points [Fig. 4]. Torsional capacity and shearing stress were recorded and tabulated for these points.

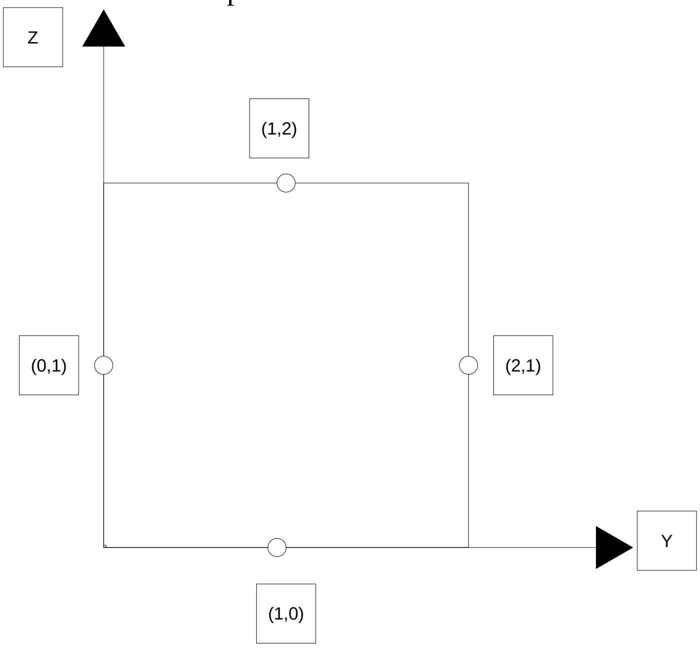


Fig. 4: Position of Recorded Shear Stress

By using Mathematica, torsional capacity and shearing stress was calculated. Table 1 shows the comparison among the cross sections.

CONCLUSIONS

In this study, torsional capacity and shear stress were found out by analysing square composite sections, which were in different arrangement of two materials (Aluminium and Brass). Two cross sections of one material (CS-I and CS-II) were analysed, which has aluminium and brass, individually. Other three cross sections (CS-III, CS-IV, and CS-V) were analysed with different combination of aluminium and brass. It was found that for single materials, brass gives higher torsional capacity and can bear higher shearing stress among the cross sections having single material (CS-I and II), because brass has higher modulus of rigidity. Composite section having brass inside and aluminium outside (CS-III), gives higher torsional capacity. Because, the core material has higher modulus of rigidity. Composite section

Table 1: Torsional Capacity, Shearing Stress

Sl. No.	Cross Section	Shearing Stress, τ (MPa)				Torsional Capacity, T (N-mm)
		(1,0)	(2,1)	(1,2)	(0,1)	
01	CS-I	32357.68	32357.68	32357.68	32357.68	53864.31
02	CS-II	53929.46	53929.46	53929.46	53929.46	89773.86
03	CS-III	40165.51	40165.51	40165.51	40165.51	72925.5
04	CS-IV	46121.63	46121.63	46121.63	46121.63	70712.67
05	CS-V	43632.4	33776.33	32932.39	33776.33	58766.93

having brass outside and Aluminium inside (CS-IV) can bear higher shearing stress. It is known that shearing stress varies with the length and increases with the increase of distance from the centre and material having higher modulus of rigidity was placed outside. Due to this fact, CS-IV can bear higher shearing stress comparing with other composite cross section. So, it can be concluded that

- Materials having high modulus of rigidity should be used for high torque, where high torsional resistance is required.
- Materials having higher modulus of rigidity should be placed outside in a cross section for the resistance of higher shearing stress optimum material area.

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