NUMERICAL STUDIES OF SHAPE EFFECT OF SQUARE AND CIRCULAR FOOTING PLACED ON COHESIVE–FRICTIONAL WEIGHTLESS MEDIUM

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ABSTRACT

In this study, three-dimensional finite element models, incorporating Mohr-Coulomb elasto-plastic material model, are validated for the evaluation of the shape effect of the square and circular surface footing under vertical loading in c-ϕ soil. The numerical models have closely predicted experimental load-settlement relationships. The shape effects on the results are also discussed in relation to the progressive failure around the foundations and the shape of the failure mechanism inside the soil. Having detailed parametric studies, the shape factors of square footing are fitted by a simple exponential function of the soil friction angle and shape factors of circular footing are expressed as a function of shape factor of square footing.

Keywords: Bearing capacity; square footing; circular footing; numerical analysis

INTRODUCTION

Bearing capacity of soil is one of the most interesting research subjects in geotechnical engineering as this problem has multi-dimensions with respect to the geometry of footing, loading and supporting foundation soil. Extensive studies were conducted for bearing capacity in two dimensions for infinitely long strip footing rest on a horizontal and inclined slope surface. In this regard, different methods of analysis and theories were developed over last few decades to determine the bearing capacity of soil. But the basic structure of formulae used for calculations of bearing capacity today, was first proposed by Terzaghi in 1943. The first important contributions are due to Prandtl (1920) and Reissner (1924), who considered a rigid perfectly plastic half space loaded by a strip punch and Sokolovski (1965), in regard to ponderable soil, all under plain strain conditions. Keverling Buisman (1940) and Terzaghi (1943) proposed the following formula to calculate the ultimate bearing pressure of soil beneath the footing, where the influence of soil cohesion (c), surcharge (q) and the weight of soil (γ) are considered independently.

$$\frac{Q_{ult}}{B} = q_{ult} = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma \tag{1}$$

Where Q_{ult}, q_{ult} = Ultimate load and pressure respectively; B= footing width; D= depth of embedment; γ=unit weight of the soil; and c=soil cohesion and N_c, N_q and N_γ=bearing capacity factors dependent only on the angle of the internal friction of soil. Terzaghi calculated all three components in Eq. (1) based on limit equilibrium. Prandtl (1920) and Reissner (1924) calculated the bearing capacity factor N_c and N_q for weightless soil using the method of characteristics assuming that the soil satisfied associate flow rule. The stress field for two independent solutions by Prandtl (1920) and Reissner (1924) has identical trajectories of principal stress and, even though the stress equation is nonlinear, the superposition of first two terms yields the correct solution. However, once the soil weight is considered, the Eq. (1) is not strictly valid, but it is used in design as a reasonable estimate. These two factors in Eq. (1) take the form

$$N_q = tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{2}$$

$$N_c = \left(N_q - 1\right)cot\phi \tag{3}$$

Where ϕ = internal friction angle. Michalowski (2001) obtained N_c directly for frictional soil by applying “rules of equivalent states”(Caquot 1934). There are several solutions in the literature for the third factor N_γ. Meyerhof (1951, 1963); Hansen (1970); Vesic (1973); Hjiaj et.al (2005); Kumar and Khatri (2008) and Chakraborty and Kumar (2013) subsequently proposed different equation to calculate this bearing capacity factor. In contrast, there are large differences among the published numerical solutions for N_γ.

In recent years, both theoretical and experimental investigation on the ultimate bearing capacity of square and circular footings received the attention of many researchers (Cerato and Lutenegger 2006; Merifield and Nguyen 2006; Cerato and Lutenegger 2007; Yu et al. 2010; Lavasan and Ghazavi 2012; Ma et al. 2014). However, according to the author knowledge, very few experimental studies (Pathak et al. 2008) were performed that estimate the bearing capacity of square and circular footing placed on c-ϕ soil. Therefore, an extensive experimental investigation is required to determine the ultimate capacity of footing on c-ϕ soil, which would be a helpful tool for the design engineers.

This paper deals with the experimental and numerical investigation of ultimate bearing capacity of c-φ soil beneath square and circular footing subjected to vertical load, exploring the differences of failure mechanism of soil under the both footings. A detailed parametric study is carried out to determine the shape effect of square and circular footing as a function of soil property. Finally a new set of equations of shape factor for square and circular footing is proposed comparing its performance with past studies.

MODEL FOOTING TESTS

Plate load test provide a direct measure of compressibility and occasionally the bearing capacity of soils. The technique adopted in this investigation for carrying out the plate loading test is described in D1194-94. (1998). The size of the square and circular model footings used were 400 mm and 420 mm, respectively, having a steel base with thickness of 30 mm. All tests were performed with the footing resting on the soil surface on the saturated clayey soils. The load was applied to the plate incrementally via a factory calibrated hydraulic load cell and a hydraulic jack, and the settlement was measured using computerized data acquisition system. In order to measure any tilt that may occur, two gauges on the perimeter of the plate were used. These gauges supported on rigid uprights fixed firmly into the ground at a distance of more than twice the plate width from the plate center. From the load-settlement data, a load settlement curve for square and circular footing was produced. The ultimate bearing capacity and the settlement of the footings were determined from load settlement curve for the test plates (Figure 2). Collecting the undisturbed samples from the soils of the test locations, following soil properties were obtained in the laboratory.

FINITE ELEMENT MODEL AND VALIDATION

Finite element engine ABAQUS was used to determine the failure load (ultimate bearing capacity) of square and circular footing. The program is most suitable for analyzing nonlinear behavior of material, failure phenomena and related instability. The three-dimensional finite-element mesh used for analysis of a circular footing and square footing of as shown in Figure 1. It represents a half-footing cut through one of the orthogonal planes of symmetry. In numerical simulations, the elastic-perfectly plastic, associative Mohr-Coulomb material model was used. The material parameters used in the analysis is given in Table 1. Eight node linear brick elements with reduced integration were used for discretization of the foundation soil. The distance between the boundaries parallel to the footing length is 15 times the width of footing and the depth of the model is half of that distance (Zhu and Michalowaski 2005). The base of soil layer is fixed in all directions. All vertical boundaries are fixed in horizontal direction but free in vertical direction. The rigid surface footing is modeled by applying uniform vertical downward displacements at all nodal points below the footing at the top surface of domain.

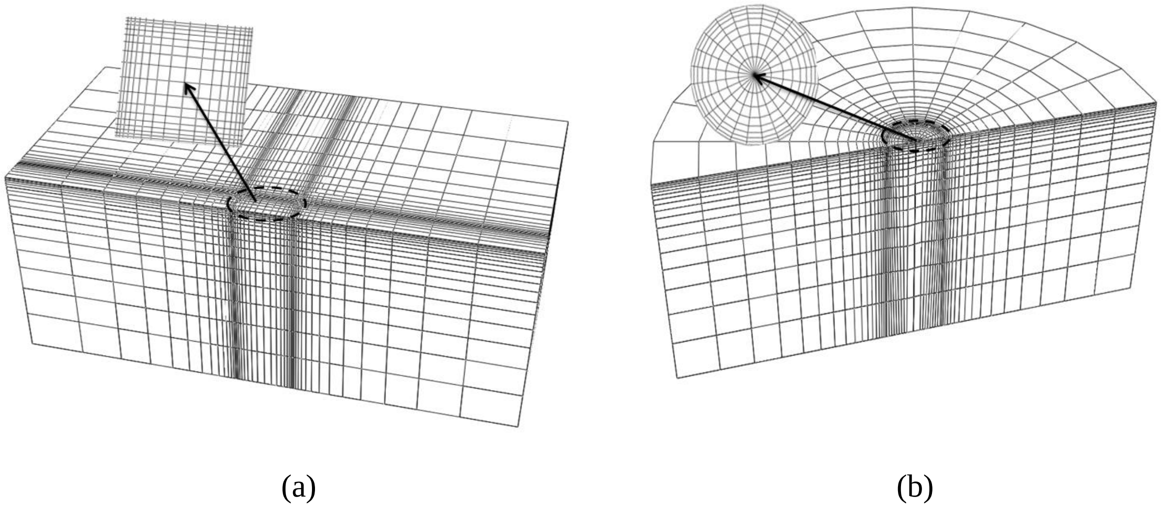

Fig. 1. Finite element meshes: (a) Half square footing and (b) Half circular footing.

Horizontal displacements at the footing-soil interface were restrained to against movement to model the perfect rough base of the footing. To determine the collapse load of the footing, displacement based analyses were performed. The total displacement was applied over a number of sub-steps and the bearing pressure was then calculated by summing the vertical components of the forces at the nodal points immediately beneath the footing divided by the footing area. The mesh is refined in the vicinity of foundation edge since it is in the zone of stress concentration. In this study, mesh convergence studies were performed to optimize the mesh size especially at the neighborhood of footing.

Table 1. Material Parameters

Parameter	Value
Bulk density, γ (kN/m^3)	15.83
Elastic modulus, E_s (kPa)	4,800
Cohesion, c (kPa)	10.75
Internal frictional angle, ϕ (Degrees)	20

RESULTS AND DISCUSSIONS

The load-displacement curves obtained from the analysis for square and circular surface footings are shown in Figure 2. It is observed that the FE model can satisfactorily predict the experimental data points. It is noticed that the settlement curves of the circular and square footing is almost same up to the settlement of 13 mm and then, they deviates. The bearing capacity of square footing is 1.21 times higher than the circular footing. This is consistent with the experimental results obtained by Terzaghi (1943) and Cerato and Lutenegger (2006), where the bearing capacity of square footing is approximately 1.33 and 1.25 times higher than a circular footing according to Terzaghi and Cerato et al, respectively. Terzaghi (1943) proposed the shape factors s_c=1.3, s_q=1, s_γ =0.8, and s_c=1.3, s_q=1, s_γ=0.6 for square and circular footing, respectively. The factors s_c=1.3, s_γ=0.8 for square footing in Terzaghi's suggestion was derived from Golder (1941)'s experiments on clay soil with 3 in. square, 18 by 3 in. rectangular and sand with 6 in. square footing. These test data were highly scattered and Terzaghi disregarded the scatter for establishing a provisional equation. Terzaghi also ignored the influence of internal friction angle on shape factors.

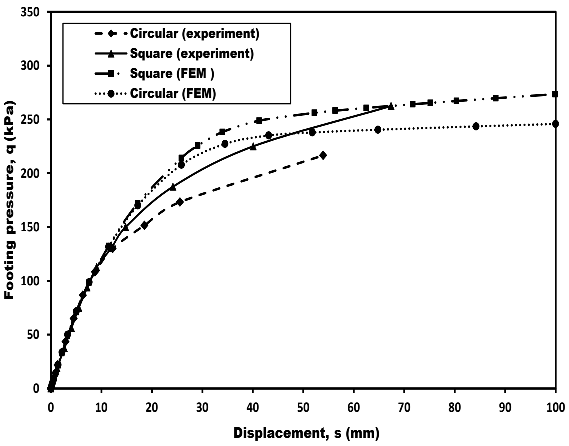

Fig. 2. Load displacement curves of square and circular footing

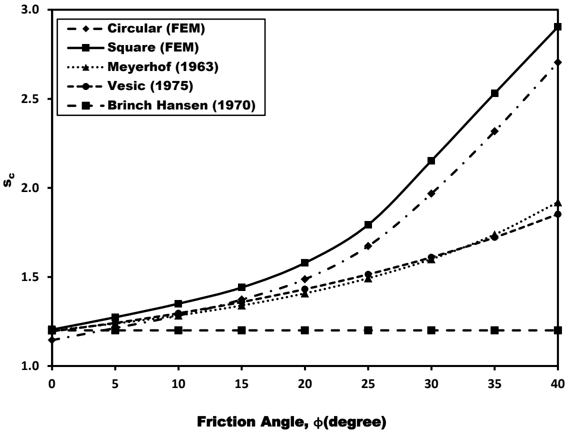

Fig. 3. Cohesion shape factor (sc) as a function of internal friction angle

Terzaghi's proposal for shape factors (s_c and s_q) of square and circular footing was same, but there was small difference in s_γ.Variation of s_c for both square and circular footing with earlier approaches (Meyerhof, 1963; Hanssen, 1970; Vesic, 1973) are shown in Figure 3. Earlier approaches that are presented here for square footing only, which are based on small size experiments or semiempirical considerations. Hence, the bearing capacity of circular footing is being considered same as that of the square footing in many design codes. Factor s_c calculated using

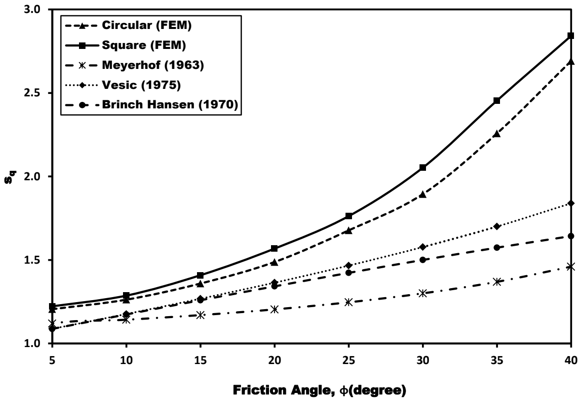

Fig. 4. Variation of surcharge shape factor (sq) as a function of friction

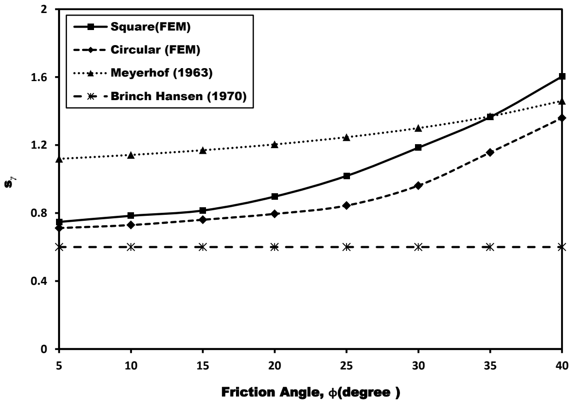

Fig. 5. Surcharge shape factor sγ as a function of friction angle

Meyerhof (1963) and Vesic (1973) methods fall very close to one another especially lower friction angles. The newly proposed s_c for square footing or circular footing is greater than Meyerhof and Vesic's solution and the differences are below 1% at ϕ=0°and increased to 30% at ϕ=40°. Zhu and Michalowaski (2005) also proved with their finite element analysis that the shape factors of Meyerhof are far too low. On the other hand, Hansen's proposal for cohesion shape factor of square footing is constant (s_c =1.2) and it is independent of friction. From experimental and numerical results, it can be concluded that Terzaghi and Brinch & Hanssen proposals for s_c are invalid.

Figure 4 presents the effect of friction angle on the shape modifier (s_q) for both square and circular footing. It indicates that the difference of s_q is found small at lower values of friction angle and it increases with the increase of friction angle. The trends that are shown are similar to the trends of s_c (Figure 3). The maximum difference between the shape factor s_c and s_q for square footing is below 5% and this difference decreases to 4% for circular footing. For this reason, s_q can be expressed as a function of s_c. But in this paper, s_c and s_q of circular footing is expressed as a function of s_c and s_q of square footing, respectively. Earlier proposals (Meyerhof 1963;, Brinch Hanssen 1970;,Vesic 1973) show conservative estimation of s_q as compared to the finite element results for both footings.

Figure 5 shows that the Meyerhof's proposal and Brinch and Hanssen's proposal for s_γ is contradictory to one another. According to the Brinch Hanssen's proposal, s_γ is constant and independent of ϕ. On the other hand, Meyerhof's proposal shows that s_γ increases with the increase of ϕ. According to this proposal $s_\gamma=1$ when $\phi=0°$, $s_\gamma=s_c$ when $\phi\geq10°$ and s_γ will never less than 1. But, Terzaghi's suggestion for shape modifiers s_γ for square ($s_\gamma=0.8$) and round ($s_\gamma=0.6$) load is less than 1. Meyerhof's proposal is quite contrary with Terzaghi's suggestion as well as numerical results of this study. The work done by the soil weight during deformation is called the effect of soil weight on bearing capacity. When soil is incompressible ($\phi=0°$), the net work will be zero, because the negative work of soil volume that moves upward is equal to the positive work of the soil volume that moves downward. In this way, the influence of soil weight on bearing capacity is negligible, $N_\gamma=0$ when $\phi=0°$ and $N_\gamma>0$ when $\phi>0°$. Erickson and Drescher (2002) and Zhu and Michalowski (2005) also proved that for small dilatancy angles the volume of displaced soil for a circular and square footing is less than the volume of displaced soil in plane-strain mechanism. But at larger dilatancy angle this relationship is opposite. For this reason, s_γ can be less than 1 as similar to the result shown in Figure 5. It shows that, s_γ for circular footing is always lower than the s_γ of square footing. s_γ for circular footing changes from 0.71 to 1.36 at a friction angle ranging from 5 to 40°. Consequently, s_γ for square footing changes from 0.75 to 1.60 at a friction angle ranges from 5 to 40°.

CONCLUSIONS

This paper presents that numerical models have predicted closely the experimental data points of the load-settlement relationships of square and circular footings under vertical loading on homogeneous clay soil. It is observed that there is a difference between their load-settlement behaviors and ultimate bearing capacities. Based on this study, it is fair to conclude that square footing exhibits higher bearing capacity as well as shape factors than the circular footing on homogeneous clay soil. Detailed parametric studies are conducted to compare the shape effect of circular and square footing as a function of frictional angle. This study has proposed new set of shape modifiers, s_c, s_q and s_γ for circular and square footings.

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