IDENTIFYING PARAMETERS OF RAINFALL INTENSITY-DURATION-FREQUENCY RELATIONSHIP: A CASE STUDY FOR NORTH-EASTERN REGION IN BANGLADESH

M. M. Rasel[1*] & S. A. Esha[2]

[1]*Department of Civil Engineering, Ahsanullah University of Science and Technology, Dhaka, Bangladesh*
[2]*Department of Water Resources Engineering, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh*
[*]*Corresponding Author: munshimdrasel@gmail.com*

ABSTRACT
The objective of this research is to identify the parameters of Rainfall IDF relationship at North-Eastern region of Bangladesh. Two common frequency analysis techniques Gumbel and Log Pearson Type III (LPTIII) distribution were used to develop the IDF relationship from rainfall data of this region. Yearly maximum rainfall data for last 41 years (1974-2014) from Bangladesh Meteorological Department (BMD) was used in this study. The results obtained using Gumbel method are slightly higher than LPT III distribution method. The chi-square goodness of fit test was used to determine the best fit probability distribution. The parameters of the IDF equations and coefficient of correlation for different return periods (2, 5, 10, 25, 50 and 100 years) were calculated by using nonlinear multiple regression method. It was found that intensity of rainfall decreases with increase in rainfall duration. Further, a rainfall of any given duration will have a larger intensity if its return period is large. In other words, for a rainfall of given duration, rainfalls of higher intensity in that duration are rarer than rainfalls of smaller intensity.

Keywords: Rainfall intensity; rainfall duration; rainfall frequency; Gumbel's extreme value distribution method; Log Pearson Type III

INTRODUCTION

Rainfall intensity-duration-frequency (IDF) curves are graphical exemplifications of the amount of water that falls within a given period of time in catchment areas (Dupont et al., 2000). IDF curves are used to aid the engineers while designing urban drainage works. The establishment of such relationships was done as early as 1932 (Chow ,1988 ; Dupont et al.,2006). Since then, many sets of relationships have been constructed for several parts of the globe. However, such relationships have not been accurately constructed in many developing countries. Koutsoyiannis et al.(1998); Koutsoyiannis et al. (2003) cited that the IDF-curves allow for the estimation of the return period of an observed rainfall event or conversely of the rainfall amount corresponding to a given return period for different aggregation times. In Bangladesh water logging and flood is a common problem during Monsoon period because of inadequate drainage system. In order to solve this problem new drainage design is needed where rainfall data of different duration is needed. But due to instrumental limitation these data were not available. In the present study, annual maximum rainfall series is considered for Rainfall Frequency Analysis (RFA). Rainfall in a region can be characterized if the intensity, duration and frequency of the diverse storms occurring at that place are known. The frequency-data for rainfalls of various durations, so obtained, can be represented by IDF curves, which give a plot of rainfall intensity versus rainfall duration and recurrence interval. In recent studies, various authors attempted to relate IDF relationship to the synoptic meteorological conditions in the area of hydrometric stations (Dupont et al. 2006; Mohymontl et al. 2004). Matin et al.(1984), in their study developed the IDF curve for North-East region Bangladesh and also observed that the rainfall data in this region follow Gumbel's Extreme Value Distribution. developed The short duration rainfall IDF curve was developed for Sylhet with return period of 2, 5, 10, 20, 50, and 100 years (Chowdhury et al. 2007). Kim et al. (2008) improved the accuracy of IDF curves by using long and short duration separation technique. They derived IDF curves by using cumulative distribution function (CDF) for the site under consideration using multi-objective genetic algorithm. Khaled et al. (2011) applied L-moments and generalized least squares regression methods for estimation of design rainfall depths and development of IDF relationships. Rashid et al. (2012) applied Pearson Type-III distribution for modelling of short duration rainfall and development of IDF relationships for Sylhet city in Bangladesh. In this context, an attempt has been made to estimate the rainfall for different return periods for different durations of 'n' such as 10-min, 20-min, 30-min, 60-min, 120-min, 180-min, 360-min, 720-min, 1440-min adopting Gumbel distributions for development of IDF relationships for North-Eastern regions of Bangladesh using updated data. Model performance indicators (MPIs) such as correlation coefficient (R) was used to analyse the performance of the developed IDF relationships by Gumbel distributions & LPTIII method for estimation of rainfall intensity of the stations under study.

METHODOLOGY

For this study 24 hour daily rainfall data from year 1974 to 2014 was collected from Bangladesh Meteorological Department (BMD) for North-Eastern region. From the daily data maximum yearly rainfall data was used in the analysis. For accurate hydrologic analyses, reliable rainfall intensity estimates are necessary. The IDF relationship includes the estimate of rainfall intensities of different durations and recurrence intervals. Two common frequency analysis techniques were used to develop the relationship between rainfall intensity, storm duration, and return periods from rainfall data for the regions under study. These techniques are: Gumbel distribution and LPT III distribution.

Estimation of Short Duration Rainfall

Chowdhury et al. (2007), used Indian Meteorological Department (IMD) empirical reduction formula to estimate the short duration rainfall from daily rainfall data in Sylhet city and found that this formula give the best estimation of short duration rainfall. In this study this empirical formula "EQ. (1)" was used to estimate short duration rainfall of six stations of Central region of Bangladesh.

$$P_t = P_{24} \sqrt[3]{\frac{t}{24}} \quad (1)$$

Where, P_t is the required rainfall depth in mm at t-hr duration, P_{24} is the daily rainfall in mm and t is the duration of rainfall for which the rainfall depth is required in hr.

Gumbel Theory of Distribution

Gumbel distribution methodology was selected to perform the flood probability analysis. The Gumbel method calculates the 2, 5, 10, 25, 50 and 100 year return intervals for each duration period and requires several calculations. Frequency precipitation PT (in mm) for each duration with a specified return period T (in year) is given by the following equation:

$$PT = Pave + KS \quad (2)$$

Where K is Gumbel frequency factor given by:

$$K = -\frac{\sqrt{6}}{\pi}\left[0.5772 + \ln\left[\ln\left[\frac{T}{T-1}\right]\right]\right] \quad (3)$$

Where Pave is the average of the maximum precipitation corresponding to a specific duration.
In utilizing Gumbel's distribution, the arithmetic average in Eq. (2) is used:

$$P_{ave} = \frac{1}{n}\sum_{i=1}^{n} Pi \quad (4)$$

Where Pi is the individual extreme value of rainfall and n is the number of events or years of record. The standard deviation is calculated by EQ. (5) computed using the following relation:

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}(Pi - Pave)^2\right]^{1/2} \quad (5)$$

Where S is the standard deviation of P data. The frequency factor (K), which is a function of the return period and sample size, when multiplied by the standard deviation gives the departure of a desired return period rainfall from the average. Then the rainfall intensity, I_T (in mm/h) for return period T is obtained from:

$$I_T = \frac{Pt}{Td} \quad (6)$$

Where T_d is duration in hours.

Log Pearson type III

The LPT III probability model is used to calculate the rainfall intensity at different rainfall durations and return periods to form the historical IDF curves for each station. In the same manner as with Gumbel method, the frequency precipitation is obtained using LPT III method. The simplified expression for this latter distribution is given as follows:

$$P^* = \log(\text{Pi}) \quad (7)$$

$$P_T^* = Pave^* + K_T S^* \quad (8)$$

$$Pave^* = \frac{1}{n}\sum_{i=1}^{n} P^* \quad (9)$$

$$S^* = \left[\frac{1}{n-1}\sum_{i=1}^{n}(P^* - Pave^*)^2\right]^{1/2} \quad (10)$$

Where P_T*, Pave*, S* are as defined previously in previous but based on the logarithmically transformed Pi values; i.e. P* of Eq. (7). KT is the Pearson frequency factor which depends on return period (T) and skewness coefficient (Cs). The skewness coefficient, Cs, is required to compute the frequency factor for this distribution. The skewness coefficient is computed by "Eq. 11"

$$C_S = \frac{n\sum_{i=1}^{n}(Pi^* - Pave^*)^3}{(n-1)(\text{n}-2)(\text{S}^*)^3} \quad (11)$$

By knowing the skewness coefficient and the recurrence interval, the frequency factor, KT for the LPT III distribution can be extracted. The antilog of the solution in Eq. (7) will provide the estimated extreme value for the given return period.

Derivation of IDF equation

The IDF formula are the empirical equations representing a relationship between maximum rainfall intensity as a dependent variable and other parameters of interest; for example the rainfall duration and frequency as independent variables. Two approaches were tried to estimate the equation parameters.

A. By applying the logarithmic conversion, where it is possible to convert the equation into a linear equation, thus to calculate all the parameters related to the equation.

B. Estimation of the equation parameters by using nonlinear regression analysis: Using the Solver function of the ubiquitious spreadsheet programme Microsoft Excel, which employs an iterative least squares fitting routine to produce the optimal goodness of fit between data and function. The R^2 value calculated is designed to give the user an estimate of goodness of fit of the function to the data.

Goodness of fit test

The aim of the test is to decide how good is a fit between the observed frequency of occurrence in a sample and the expected frequencies obtained from the hypothesized distributions. A goodness of fit test between observed and expected frequencies is based on the chi-square quantity, which is expressed as "Eq. 12"

$$\chi^2 = \sum_{i=1}^{k} (\mathrm{O_i} - \mathrm{E_i})^2 / \mathrm{E_i} \qquad (12)$$

Where χ^2 is a random variable whose sampling distribution is approximated very closely by the chi-square distribution. The symbols Oi and Ei represent the observed and expected frequencies, respectively, for the i-th class interval in the histogram. The symbol k represents the number of class intervals. If the observed frequencies are close to the corresponding expected frequencies, the χ^2 value will be small, indicating a good fit; otherwise, it is a poor fit. A good fit leads to the acceptance of null hypothesis, whereas a poor fit leads to its rejection. The critical region will, therefore, fall in the right tail of the chi-square distribution. For a level of significance equal to a, the critical value is found from readily available chi-square tables and $\chi^2 >$ constitutes the critical region.

RESULTS AND DISCUSSIONS

According to the IDF curves, rainfall estimates are increasing with increase in the return period and the rainfall intensities decrease with rainfall duration in all return periods. Rainfall intensities rise in parallel with the rainfall return periods. The results obtained from the two methods have good consistency.

Table 1: The parameters values used in deriving formula

Region	Parameter	Gumbel Method	Log Pearson III Method
North-Eastern	c	759	664
	m	0.268	0.255
	e	0.667	0.666

Table 1. shows the parameters values obtained by analyzing the IDF data using the two methods and those are used in deriving formulae for the two regions. Fig.1 and Fig.2 show results of the IDF curves obtained by Gumbel and LPT III methods for North-Eastern region. It was shown that there were small differences between the results obtained from the two methods, where Gumbel method gives slightly higher results than the results obtained by Log Pearson III. This is shown also from parameters of the derived equation for calculating the rainfall intensity using the two methods.

Parameters of the selected IDF formula were adjusted by the method of minimum squares, where the goodness of fit is judged by the correlation coefficient. The results obtained showed that in all the cases the correlation coefficient is very high, and ranges between 0.998 and 0.987, except few cases where it ranges between 0.986 and 0.978 when using LPT III at 50 and 100 years. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest. For each region the results are given as the mean value of the points results.

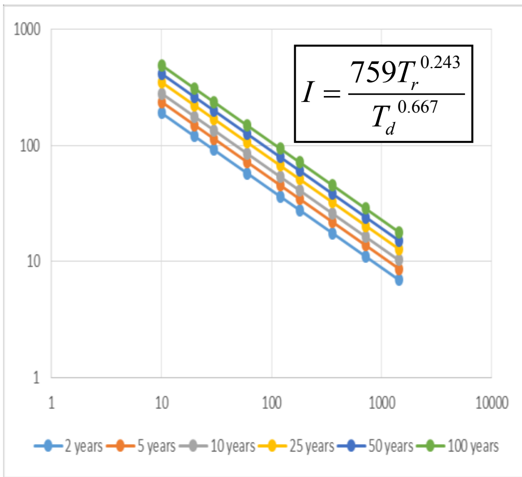

Fig.1: IDF curve by Gumbel method

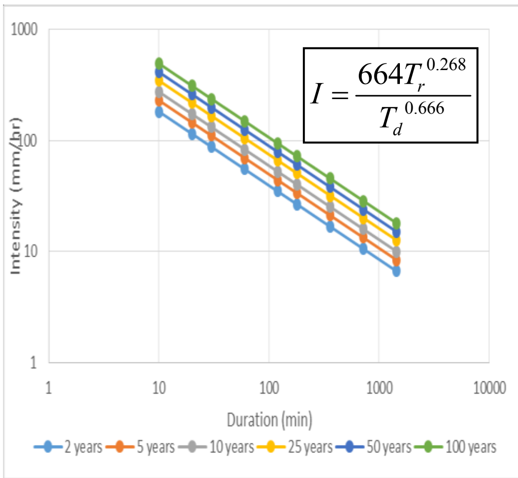

Fig.2 : IDF curve by LPTIII method

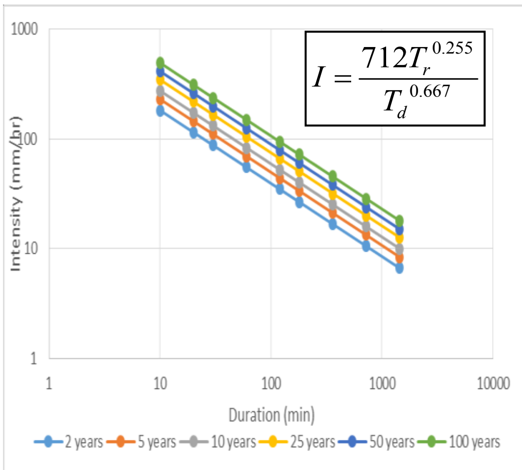

Fig.3 : IDF curve by average

Also,goodness-of-fit tests were used to choose the best statistical distribution among those techniques. Results of the chi- square goodness of fit test on annual series of rainfall are shown in Table 2. As it is seen most of the data fit the distributions at the level of significance of =0.05, which yields <3.84. Only the data for 10 min do not give good fit using Gumbel method distribution. Also the data for 10, 20, 30 min using LPTIII method do not give good fit.

Table 2: Results of chi-square goodness of fit test on annual maximum rainfall

Region	Distribution	Duration (min)								
		10	20	30	60	120	180	360	720	1440
North-Eastern	Gumbel	5.54	3.49	2.66	1.67	1.0	0.80	0.50	0.319	0.201
	LPTIII	8.24	5.19	3.95	2.49	1.5	1.1	0.7	0.47	0.298

CONCLUSIONS

Since Bangladesh has different climatic conditions from region to region, a relation for each region has to be obtained to estimate rainfall intensities for different durations and return periods ranging between 2 and 100 years. It was found that Gumbel method gave some larger rainfall intensity compared to LPT III distribution. In general, the results obtained using the two approaches are very close at most of the return periods and have the same trend. The parameters of the design storm intensity for a given period of recurrence were estimated for each region. The results obtained showed a good match between the rainfall intensity computed by the methods used and the values estimated by the calibrated formulae. The results showed that in all the cases data fitted the formula with a correlation coefficient greater than 0.97. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest for durations varying from 10 to 1440 min and return periods from 2 to 100 years. The chi-square test was used on one hand to examine the combinations or contingency of the observed and theoretical frequencies, and on the other hand, to decide about the type of distribution which the available data set follows. The results of the chi-square test of goodness of fit showed that in all the durations the null hypothesis that the extreme rainfall series have the Gumbel distribution is acceptable at the 5% level of significance. Only few cases in which the fitting was not good obtained by using the LPT III distribution. Although the chi-square values are appreciably below the critical region using Gumbel distribution and few values are higher than the critical region using LPT III distribution, it is difficult to say that one distribution is superior to the other. Further studies are recommended whenever there will be more data to verify the results obtained or update the IDF curves.

ACKNOWLEDGMENTS
Authors are indebted to the Bangladesh Meteorological Department (BMD) for providing the necessary meteorological data.

REFERENCES

Chow, VT. 1988. Handbook of Applied Hydrology. McGraw-Hill Book.

Dupont, BS.; Allen, DL. 2006. Establishment of Intensity–Duration–Frequency Curves for Precipitation in the Monsoon Area of Vietnam. Kentucky Transportation Center, College of Engineer, University of Kentucky in corporation with US Department of Transportation.

Haddad, K.2011.Design rainfall estimation for short storm durations using L-moments and generalised least squares regression-application to Australian data. *Int J Water Res Arid Environ* 1 (2011): 210-218.

Kim ,T; Shin, J.; Kim , K. and Heo, J. 2008. Improving accuracy of IDF curves using long- and short duration separation and multi-objective genetic algorithm. World Environmental and Water Resources Congress. 2008, 1-12.

Koutsoyiannis, D ; Demosthenes K and Alexandros M. 1998. A mathematical framework for studying rainfall intensity-duration-frequency relationships. Journal of Hydrology 206.1 (1998): 118-135.

Koutsoyiannis, D. 2003. On the appropriateness of the Gumbel distribution for modelling extreme rainfall. Proceedings of the ESF LESC Exploratory Workshop held at Bologna. 2003.

Matin , MA and Ahmed SMU.1984. Rainfall Intensity Duration Frequency Relationship for the N-E Region of Bangladesh. *Journal of Water Resource Research.* 5(1).

Mohymont1,B .; Demaree1,GR. and Faka2,DN. 2004. Establishment of IDF-curves for precipitation in the tropical area of Central Africa-comparison of techniques and results. Department of Meteorological Research and Development, Royal Meteorological Institute of Belgium, Ringlaan 3, B-1180 Brussels, Belgium.

Rashid, M M.; Faruque, SB and Alam, JB. 2012. Modeling of short duration rainfall intensity duration frequency (SDRIDF) equation for Sylhet City in Bangladesh. *ARPN Journal of Science and Technology* 2.2 (2012): 92-95.