Proceedings of the
International Conference on Mechanical Engineering and Renewable Energy 2015
(ICMERE2015) 26 – 29 November, 2015, Chittagong, Bangladesh

ICMERE2015-PI-275

APPLICATION OF RAND ALGORITHM IN INVENTORY REPLENISHMENT FOR A TEXTILE INDUSTRY

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Abstract*-In this research a suitable joint replenishment policy is studied for providing a better replenishment policy in case of multi-product, single supplier situations for chemical inventories in textile industry. It is focused on finding out the optimum ideal cycle time and individual cycle time of each product for replenishment that will cause lowest annual holding and ordering cost, and also find the optimum ordering quantity. For its expediency and ease of application an algorithm by Kaspi and Rosenblatt (1991) called RAND was used which offers an ideal cycle time (T) for replenishment and an integer multiplier (ki) for each individual item. Therefore the replenishment cycle time for each product is found as T×ki. This research uses indirect grouping strategy (IGS) and according to which, a replenishment is made at regular time intervals and each product has replenishment quantity sufficient to last for exactly an integer multiple of regular time interval.*

Keywords: Inventory Replenishment, RAND Algorithm, Joint Replenishment Problem, Textile Industry

1. INTRODUCTION

The Study of inventory problems dates back to 1915, when F. N. Harris developed a very simple however useful model of an inventory problem [1]. Inventory is the total amount of goods or materials contained in a store or factory at any given time. Almost every developing nation has textile manufacturing which is among the first industries to be established. The textile industry is primarily concerned with the production of yarn, and cloth and then the subsequent designing or manufacture of clothing and their distribution. A textile industry has a large variety of inventories that are critical to the overall profit of the firm and managing those are as a result very crucial. Inventories that are maintained by textile industries in general are (1) Raw materials (2) Finished fabric (3) Spare parts and (4) General store.

Inventory control involves maintaining a certain level of raw materials with safety stock for production that can response with the changing demands. For replenishing two way generally considered- (1) Independent replenishment (2) Joint replenishment (JRP).

Independent replenishment means replenishment of a single item from a single supplier. Whereas, joint replenishment refers to replenishment of multiple items from a single supplier.

Several algorithms have been suggested to find the solution for JRP. Kaspi and Rosenblatt in 1991, suggested a simple procedure, RAND, for determining the economic ordering quantity for items jointly replenished and their ordering frequency with respect to an ideal cycle time [2].In this re-search this algorithm has been followed to offer a suitable replenishment policy with replenishment cycle and ordering quantity. RAND is a mathematical programing method, with multi stage programming.

Because of the major ordering cost, using group replenishment may lead to substantial cost savings. The savings from group replenishment are significant when the major ordering cost is higher. Tactics to solving the JRP can be classified into two types: A direct grouping strategy (DGS) and an indirect grouping strategy (IGS). Under DGS, products are partitioned in predetermined number of sets and the products within the each set are jointly replenished with same cycle. Under IGS, a replenishment is made at regular time intervals and each product has replenishment quantity ample to last for exactly an integer multiple of systematic time interval. Groups in IGS are indirectly formed by products having the same integer multipliers. It is suggested that IGS outperforms DGS when major cost is high. This research includes the review of previous works on indirect grouping strategies.

Goyal and Shu introduced methods to find an inventory replenishment policy which produced sub-optimal solutions [3-4]. In 1974, Goyal developed an algorithm to find the optimal solution to minimize the total cost. He presented a systematic and enumerative method for determining the basic cycle time [5]. But his solution may have been computationally prohibitive for large problems. In addition, Andres and Emmons also pointed out that although it may be optimal to replenish each item at equal time intervals, it need not to be optimal to have replenishment cycle of equal time durations. They proved their contention with the help of an example, where the ratio between si/S was 50 (S=major ordering cost, si=minor ordering cost) [6].

In 1976, Silver introduced a simple heuristic to find the optimal or near optimal set of ki, (frequency of ordering of product/multiple integer). His method then uses this ki, to find the optimal or near optimal cycle time T for minimal TC (total cost) [7]. Goyal and Belton published a modification of Silver's method which gave closer to optimal solution in most cases. He has given a modified formula for attaining ki [8]. In 1988, Goyal published a method which improved upon Goyal (1974), Goyal and Belton and Silver [9]. A complete review of early work is found in a study published by Goyal and Satir [10].

Additional work by Kaspi and Rosenblatt resulted in an algorithm called RAND which was an improvement over their own study performed in 1983 [2], [11].

Silver's heuristic became a part of new heuristic algorithm proposed by Kaspi and Rosenblatt. Kaspi and Rosenblatt used the formula of determining the value of T (cycle time) from Silver's algorithm with a small modification of denoting item 1 with the lowest value of (S+si)/Divi. Their algorithm may be viewed as a local improvement of Silver's in finding the ki. RAND was improvement over this algorithm by determining minimum and maximum values for T, T_{min} and T_{max}. The authors did extensive experiment in order to compare their result with others. They concluded that RAND was an improvement over all other strategies and was almost as good as the optimal solution.

Goyal and Deshmukh modified RAND [12] and introduced a new lower bound on T_{min}. The result of testing the modified RAND on 48,000 randomly generated problems, in accordance to the authors paper, their modified RAND showed a 3.5% increase in the number of problems that achieved the optimal when compared to RAND. Van Eijs argued that for problems with low values of the major ordering cost relative to the minor cost, the strict cycle policies may result in higher total cost than the cyclic policies. Because of that he introduced a new algorithm which includes new formula to find the lower bound of T and obtained optimal results [13].

Viswanathan proposed an algorithm used for both strict cycle and cycle policies in 1996 [14]. The main focus was on providing tighter bound on T. This algorithm has shown to be computationally more efficient than the method proposed by Van Eijs. In 1994, Hariga developed two heuristic for solving JRP [15]. Both procedures start with ordering frequencies that are obtained from relaxed version of JRP in which multipliers need not be integer. In the first heuristic, some products are placed in a set, for which the integer multipliers have a value of one. At each iteration, the cycle time is reduced so that one product not in set has its ordering frequency increased. The second algorithm is a modified version of Goyal's algorithm in which the starting frequencies are derived from the solutions to the RJRP. The performance of both heuristics is tested using seven examples from the literature. Both heuristics converged to the optimal solutions. Ben-Daya and Hariga conducted a numerical experiment to test the performance of Hariga's second heuristic against Goyal and Deshmukh's modified RAND [16]. The experiment was performed on 24,000 randomly generated problems from uniform distributions with four values for the major ordering cost and six values for the number of products. Hargia's algorithm gave lower total cost for 86.9% of the problems. In addition, Hargia's algorithm was 21 times faster for 10-product problems and 40 times faster for the 20-product problems.

The rest of the research is organized as follows- Section 2 discusses the methodology and in section3, a complete definition of the problem and mathematical models needed to surface with a solution is delivered. Section4 and section5 are structured with the trial results in addition to the data analysis and cost calculations. Finally, in Section 6, some suggestions for future research is provided along with the conclusion.

2. METHODOLOGY

Chemicals are used in large quantities in the manufacture of textiles. Some of which are injurious to human well-being and the environment and, for example, may cause allergic reactions. Because of their versatile nature, chemical raw materials need special attention for optimization.

There are several types of model dealing with multi-product environments. The objective of these models is usually to minimize the total cost which is composed of two parts- (1) The setup or ordering cost; This is cost for installing the machines and equipment before production and for the products that are ordered from a supplier, this is the cost of preparing and receiving the order and the transportation cost. The cost of placing an order for different products has two components- (i) Major ordering cost independent of the number of different products in order and (ii) Minor ordering cost which depends on the number of different products in the order. (2) The holding cost; the cost of holding inventory which includes the cost of capital tied up inventory, taxes and insurances.

In this research RAND is used to determine the replenishment cycle and ordering quantity. We conducted an extensive simulation study. The basic idea of RAND is to divide the range of cycle time (T_{max}, T_{min}) into different equally spaced values of cycle time, T, and for each value of T the procedure is applied repeatedly and the cycle time with the lowest cost is selected.

Simulation based optimization methods although fairly handy for inventory optimization, still experience a number of challenges. As the simulation provides black box function which evaluate outputs for given input values. There are no analytical expression typifying input output relationship. In spite of this, simulation provides a remarkable solution. That is why a simulation on MATLAB was used to run this model while determining the optimum solution.

3. PROBLEM DEFINITION AND MODELING

The problem addressed in this research, called the inventory replenishment problem, is that of determining the replenishment cycle of ‘n’ different items that come from the same supplier as well as defining a proper demand for each item in order to minimize the total cost in due course. In these companies, the demand for each item is estimated. Then, based on these estimates, the inventory and planning department proceed to material planning. This results to two potential bad consequences, which ultimately causes increased production cost. For some cases, due to faulty or non- scientific demand forecast and replenishment policy, a large amount of excess raw material inventory is left off. On the other hand, a non-methodical forecast may also cause raw material shortage. In this study, we plan to suggest the optimal inventory replenishment strategy along with a scientific method of forecasting demand of chemical raw materials in a joint replenishment policy for the textile industry under consideration such that the current superfluous cost over chemical raw material replenishment is minimalized. The costs are (1) holding cost for on hand inventories, (2) major ordering cost (fixed cost associated with each order) and (3) minor ordering cost (variable cost incurred with each item included in the order).

Assumptions

The assumption that holds the most importance is that the replenishment policy currently pursued by textile industries is considered to be one which happens to be an individual replenishment for each item. This assumption is made as most of the textile industries do not follow any sort of scientific and systematic method of replenishment rather they highly depend on managerial decisions. A few other basic assumptions that are made are listed below.

a. Supply is readily available
b. Demand for each item is known and constant
c. No quantity discounts
d. No budget limitations on the amount of an order
e. No stock-outs are allowed
f. No limit on the amount of storage available

3.1 Equations

For IGS the cycle time for every product is an integer multiple k_i of replenishment cycle time T. Thus the cycle time for product i is

$T_i = k_i T$ ……………………………………………… (1)

And the ordering quantity for product i is

$Q_i = T_i D_i$ ……………………………………… (2)

The total annual holding cost are

$C_H = \sum_{i=1}^{n} Q_i h_i / 2 = \frac{T}{2} \sum_{i=1}^{n} k_i h_i D_i$ ……………....... (3)

The total annual ordering cost are

$C_O = S/T + \sum_{i=1}^{n} s_i / k_i T = \frac{1}{T}\left(S + \sum_{i=1}^{n} s_i / k_i\right)$ …..... (4)

The total annual cost is

$TC = C_O + C_H$

$= \frac{T}{2} \sum_{i=1}^{n} k_i h_i D_i + \frac{1}{T}\left(S + \sum_{i=1}^{n} s_i / k_i\right)$ ……….. (5)

3.2 RAND Algorithm

Step 1: Determine T_{min} and T_{max} where

$T_{min} = MIN_i\,(2s_i/D_i h_i)^{½}$ ……………………….... (6)

$T_{max} = [2(S + \sum_{i=1}^{n} s_i)/\sum_{i=1}^{n} h_i D_i)]^{½}$ …………..... (7)

Step 2: Divide the range (T_{min} and T_{max}) into ‘m’ different equally spaced values of T (T_1, T2.., T_j)
Where j= 1, 2...m
Step 3: For each potential value of T (T_1, T_2 …Tj), find all products k_i^2

$K_i^2 = 2s_i/h_i D_i T_j^2$ …………………………………. (8)

Here i= 1, 2…… n
Step 4: For Given T(T_j) find ki^* for each product i, where

$k_i^* = L \quad if\ L\ (L-1) < K_i^2 \le L\ (L+1)$ ……………….(9)

Step 5: Calculate the cycle time T* and TC, where

$T^* = [2(S + \sum_{i=1}^{n} s_i / k_i)/\sum_{i=1}^{n} k_i h_i D_i]^{½}$ …….… (10)

and $TC = \frac{T*}{2} \sum_{i=1}^{n} k_i h_i D_i + \frac{1}{T*}\left(S + \sum_{i=1}^{n} s_i / k_i\right)$... (11)

Step 6: Finally find the lowest TC and it corresponding values of T* and ki^*

3.3 Efficiency of RAND Algorithm

Kaspi and Rosenblatt in their 1991’s paper did a simulation study [2] in which they compared six algorithms which are abbreviated as follows: G73 (Goyal 1973 a, b); S76 (Silver 1976); GB79 (Goyal and Belton 1979); KR (83) (Kaspi and Rosenblatt 1983); G88 (Goyal 1988); RAND. These algorithms are compared with the optimal (full enumeration) solution (OPT). Different sets of data were considered and for each set 24000 different examples were solved and compared. In the first set of data, the demand for each item was generated from a uniform distribution between 100 and 100 000; the minor ordering (set-up) cost for each item was generated from a uniform distribution between 0.5 and 5; and the holding cost for each item was generated from a uniform distribution between 0.2 and 3.0. Six values of n were considered (n = 5, 10, 15, 20, 25, 30) and four different values of S, the major set-up cost, were considered (S= 5, 10, 15, 20). Thus 24 different settings were considered (combinations of n and S) and for each setting 1000 randomly generated problems were considered. Thus a total of 24 000 different examples were solved for this set of data. A summary of results is presented in figure 3.1.

Analyzing the data in figure 3.1, it is clearly seen that RAND dominates all previous approaches that were considered. RAND is superior to the other previous and is almost as good as the optimal solution.

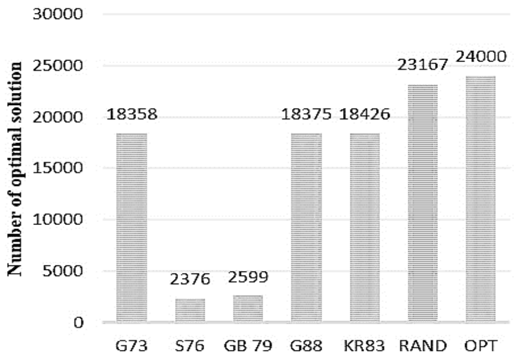

Fig.3.1: Bar chart for comparison of optimal solutions between algorithms

4. EXPERIMENTAL INVESTIGATION

Based on the data collected from a textile industry, we first performed a data analysis and converted those raw data such that they can be used as inputs. Monthly demands for each item is shown in the following tables in kilograms. $D_i$is the demand per item per year which is shown in the bottom row. Holding cost per item per year, h_i, is calculated from the monthly holding costs that have been collected from the textile industry. It was learned that minor ordering cost for each item i.e. s_i, is between 15-25 percentage of the major ordering cost for this specific industry and the major ordering cost of each replenishment is \$282.05. Random numbers (RN) was thus used to calculate the minor ordering costs. All costs taken into account are in USD and demands are in KGs. Among all, two item's yearly calculation are shown below in table 4.1. And summary of all item's yearly demand, holding and minor cost are shown in table 4.2.

Table 4.1: Input data for item no 523, 565

Item no.	523				565			
	h_i	RN	s_i	D_i	h_i	RN	s_i	D_i
	\$	%	\$	kg	\$	%	\$	kg
Jan				40190				4085
Feb				39390				7925
Mar				38240				11043
Apr				36500				12363
May				34860				19360
June	1.34	0.19	53.59	32300	4.58	0.16	45.13	12400
July				29800				7684
Aug				29200				3116
Sep				27400				6610
Oct				26600				5426
Nov				25800				7814
Dec				24600				2616
Total				384880				100442

4.1 Cost Analysis with Current Procedure

The total annual holding costs are, $C_H = \Sigma D_i h_i/2$
No. of replenishments = R_i

Total minor ordering cost per year = $\Sigma R_i s_i$

Table 4.2: Cost calculation for current procedure of replenishment

Item no.	D_i	h_i	$D_ih_i/2$	R_i	s_i	$R_i s_i$
	KG	\$	\$		\$	\$
523	384880	1.34	19928.7	0	53.59	0.00
565	100442	4.58	4939.11	10	45.13	451.28
601	34908	5.80	6174.72	4	56.41	225.64
602	21975	10.08	8830.5	1	53.59	53.59
604	87635	7.77	22065.9	8	67.69	541.54
606	37640	1.26	1915.1	3	45.13	135.38
607	156150	4.20	23539.3	4	67.69	270.77
609	294450	2.39	26035	3	45.13	135.38
610	1408000	0.67	40320	6	56.41	338.46
612	166680	6.85	37756.8	5	42.31	211.54
614	41590	4.20	4896.5	2	53.59	107.18
621	101000	5.25	19674.6	9	56.41	507.69
622	82490	4.49	1176.9	8	45.13	361.02
626	63290	4.33	10112.4	7	50.77	355.38
627	18030	4.62	3359.85	3	53.59	160.77
631	3680	14.20	849.6	3	53.59	160.77
632	3800	14.20	849.6	3	59.23	177.69
647	37160	13.15	14608	3	67.69	203.08
648	174100	4.70	27350.7	7	67.69	473.84
Sum(Σ)			**274383.28**			**4871.00**

So, the total cost of holding for 1 year is \$274383.28 and total minor ordering cost is \$4871.00 both of which after totaling with major ordering cost for 12 replenishments (\$282.05×12) in 1 year causes a total cost of \$2,82,639 for replenishment of chemical raw materials.

4.2 Cost analysis with RAND Algorithm

T_{min}= 0.0086 and T_{max}= 0.0177

Table 4.3:Total cost and optimum cycle time for different values of T_j(where, m=10)

	1	2	3	4	5	6	7	8	9	10
T_J	0.0086	0.0096	0.0106	0.0116	0.0127	0.0137	0.0147	0.0157	0.0167	0.0177
T^*	0.0101	0.0108	0.0137	0.0141	0.0147	0.0152	0.0154	0.0157	0.0161	0.0164
TC (\$) 1.0E+05 ×	1.5499	1.5363	1.4719	1.4625	1.4502	1.4450	1.4445	1.4445	1.4461	1.4484

Here T_7 and T_8 presents the lowest value for TC which is **\$1,44,450.**

Table 4.4: Optimum cycle time, total cost and integer multiplier of each item for T_7 and T_8

Item no.	T_7			T_8		
	T^*	TC (\$)	K_i^*	T^*	TC (\$)	K_i^*
523			1			1
565			1			1
601			2			2
602			2			1
604			1			1
606			3			3
607			1			1
609			1			1
610			1			1
612	0.0154	1.0e+05 ×1.4445	1	0.0157	1.0e+05 ×1.4445	1
614			2			2
621			1			1
622			1			1
626			1			1
627			2			2
631			3			3
632			3			3
647			1			1
648			1			1
Sum(Σ)			29			28

T_8 is chosen to be the optimum cycle time for having the lowest number of replenishments among the lowest TCs. The T^* of T_8 will thus be chosen as optimum for this situation.

It is seen from repeated simulation of the RAND algorithm that this set of data provides a fixed lowest value of TC within the range of m=10 to m=50. The following chart shows the values of cycle time for individual items, T_i, along with their replenishment quantity, Q_i, in correspondence to the lowest value of TC. The equations for T_i and Q_i are as stated below

$$T_i = T^* \times K_i^* \quad (12)$$

$$Q_i = T^* \times K_i^* \times D_i \quad (13)$$

Table 4.5: Individual replenishing cycle time and order quantity of each item

Item no.	TC = 144450 \$				
	K_i^*	T^*	T_i^*	D_i (KG)	Q_i (KG)
523	1		0.0157	384880	6043
565	1		0.0157	100442	1577
601	2		0.0314	34908	1096
602	1		0.0157	21975	345
604	1		0.0157	87635	1376
606	3		0.0471	37640	1773
607	1		0.0157	156150	2452
609	1		0.0157	294450	4623
610	1		0.0157	1408000	22106
612	1	0.0157	0.0157	166680	2617
614	2		0.0314	41590	1306
621	1		0.0157	101000	1586
622	1		0.0157	82490	1295
626	1		0.0157	63290	994
627	2		0.0314	18030	566
631	3		0.0471	3680	173
632	3		0.0471	3800	179
647	1		0.0157	37160	583
648	1		0.0157	174100	2733

5. RESULT

Previously, calculations of total cost is shown under the current demand of the products generated by the company. Total cost includes holding cost, minor ordering cost and major ordering cost. These costs were calculated for a duration of one year. For their current procedure of replenishment, the total cost is around \$ 2,82,639.

As the proposed algorithm is RAND. Total cost incurred using RAND was calculated in section 4.2. The total cost was calculated for different values of m (10-50) and acquired the same minimum total cost for every m. The total cost was found to be around \$ 1,44,450. The cost is reduced by approximately 49%.

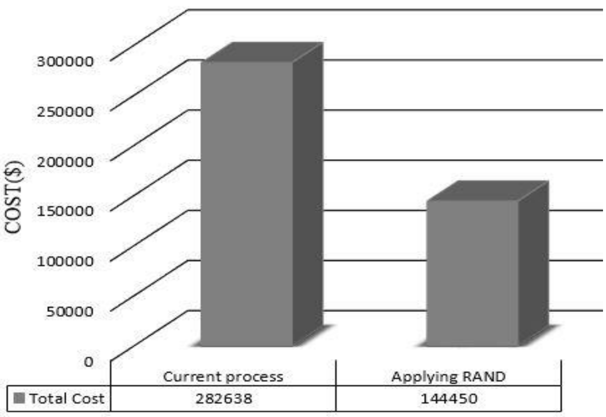

Figure 5.1: Comparison of total cost between current procedure and applying RAND

6. CONCLUSION

This research presents the inventory replenishment problems, where joint replenishment was considered as a replenishing policy. The work was done on the basis of present textile industrial environment and the purpose of this research was to introduce more systematic and organized replenishment procedure for textile industries which will minimize their annual holding and ordering costs.

In previous section, cost comparison has shown an extremely large difference where savings was about 49%. However, in the real world this cost will differ from the aforementioned costs.

In section 3, where it was described under assumption that the industry replenishes its items individually, whereas, in actuality they don't. It is true that most textile industries in Bangladesh doesn't follow a systematic replenishing policy. Here, even in case of items from the same supplier, some items are replenished individually some are done in groups. This grouping depends on managerial decisions made earlier of the replenishment. So, the replenishment process relies on the perfectionism of decision maker. That is why for ease of calculation it was assumed that all product are replenished individually and therefore the cost was found comparatively higher. Despite this fact, this research undoubtedly exhibits that application of RAND will enormously improve the replenishment policies of textile industries in Bangladesh.

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8. NOMENCLATURE

Symbol	Meaning	Unit
Y_t	Observed demand data	(Kg)
S	Major ordering cost/order	($)
s_i	Minor ordering cost/order of item i	($)
D_i	Annual demand of item i	(Kg)
v_i	Unit variable cost of item i	($)
T	Replenishment cycle time	(Year)
T_{min}	Minimum replenishing cycle time	(Year)
n	Number of items	Dimensionless
i	Product index (1, 2…n)	Dimensionless
h_i	Annual holding cost of item i	($)
Q_i	Ordering quantity of item i	(Kg)
T_i	Replenishing cycle time of item i	(Year)
k_i	Integer multiplier of item i	Dimensionless
TC	Total cost	($)
Co	Annual ordering cost	($)
C_H	Annual holding cost	($)
L	Value with integer number	Dimensionless
T_j	Time cycle of j^{th} number	(Year)
T^*	Optimum cycle time	(Year)
T_{max}	Maximum replenishment cycle time	(Year)
m	Number of equally spaced values	Dimensionless
RN	Random number	Dimensionless
R_i	Number of replenishment	Dimensionless