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EFFECT OF DISCRETE HEATER IN A TRIANGULAR CAVITY FILLED WITH NON-NEWTONIAN POWER LAW FLUID

Tawfiqur Rakib[1,*], Sourav Das[2], Khan Md. Rabbi[3], Satyajit Mojumder [4] and Sourav Saha[5]

[1-5]Department of Mechanical Engineering, Bangladesh University of Engineering & Technology
[1,*]tawfiq1448@gmail.com, [2]dsourav46@yahoo.com, [3]khanrabbi92@gmail.com, [4]sjit018@gmail.com, [5]souravsahame17@gmail.com

Abstract*- In this study, natural convection in a triangular cavity filled with non-Newtonian power law fluid is analyzed for three different cases of heater length (λ= 0.25, 0.5, 1) at the vertical wall. Numerical simulation is carried out for wide range of Rayleigh numbers ($Ra = 10^3 \sim 10^6$) and power law index (n = 0.5, 1, 1.5). Galerkin weighted residuals method of finite element analysis is adopted in this study and numerical accuracy is ensured by grid independency test and code validation. Results are shown in terms of streamlines, isothermal lines and Nusselt number (Nu) plots. Result of this investigation indicates that heat transfer rate decreases with the increment of power-law index for any length of heater. It is concluded that increment of heater length increases the heat transfer rate in a significant manner. Thus more than 42% better heat transfer rate is obtained for heater length (λ = 1.0).*

Keywords: Discrete heater, Rayleigh number, Power-law index, Non-Newtonian fluid

1. INTRODUCTION

Natural convection in enclosed cavities has drawn interest of many researchers all over the world [1-2]. Among different types of enclosures, triangular enclosures have received a considerable attention because of its numerous applications in various fields of engineering such as: (a) Building and thermal insulation systems [3-4] and (b) Geophysical fluid mechanics [5-6].

The flow of non-Newtonian power law fluids has wide potential application in many areas of engineering e.g. (a) In the process industries [7] (b) Peristaltic transport of a non-newtonian fluid: Applications to the vas deferens and small intestine. [8] Several researches on natural convection for non-Newtonian power law fluids were conducted by researchers for different geometric shaped models and numerical methods [9-10]. An extensive review of non-Newtonian fluid flow through non-circular ducts was reported by Lawal et al. [11]. Etemad et al. [12] conducted Galerkin finite element analysis to solve the fully developed non-Newtonian flow problem through triangular channels. Hashemabadi et al. [13] studied laminar flow of non-Newtonian power law fluid in right triangular ducts by a correlation for prediction of friction factor for different aspect ratios and power law index. Results indicated that, power law index had significant influence on flow behavior. Lamsaadi et al. [14] performed the study of natural convection in a vertical rectangular cavity filled with non-Newtonian fluid and subjected to uniform heat flux along the vertical walls numerically. They performed their result with Newtonian case and concluded that shear thinning behavior increments heat transfer rate of the fluid flow and on the other hand for shear thickening fluid, it decreases. Matin et al. [15] examined two dimensional steady-state natural convection of power-law fluids numerically between two concentric horizontal cylinders with different constant temperatures. They revealed that the non-Newtonian fluids were more feasible than Newtonian fluids for cooling and insulating purposes. It was shown that with the increment of Rayleigh number the cooling effect of pseudo-plastic fluid and the insulating effect of dilatant fluid became more effective.

Sojoudi et al. [16] numerically investigated free convection heat transfer in a differentially heated trapezoidal cavity filled with non-Newtonian Power-law fluid. They studied nominal Rayleigh numbers for $Ra = 10^5 – 10^7$, Prandtl numbers of Pr = 100 – 10000 and power-law index (n) from 0.6 to 1.4. Their result concluded that, Pr variation had no significant effect on Nu as most non-Newtonian fluids contain higher values of Pr and also maximum stream function is reduced by the increase of Pr. Turan et al. [17] simulated two-dimensional steady-state of laminar natural convection in square enclosures with differentially heated sidewalls subjected to constant wall temperatures where the enclosures were considered to be completely filled with non-Newtonian fluids. They proved that the average Nusselt number increased with increment of values of the Rayleigh number for both Newtonian and power law fluids. Moreover, the simulation results displayed that the average Nusselt number was marginally affected by the increase in Prandtl number for Newtonian and power law fluids for a given set of values of the

Rayleigh number and power law index.

The main aim of our study is to investigate the effect different heater length (λ = 0.25, 0.5, 1) at the vertical wall of a triangular cavity on natural convection heat transfer numerically. Non-Newtonian power law fluid is considered as the working fluid. Parametric studies have been performed by varying different pertinent parameters Rayleigh number (*Ra*) and power-law index (*n*).

2. PROBLEM SPECIFICATION

The geometry of the present problem is shown in Fig.1. It consists of a two-dimensional right angled triangular cavity. The temperature of the enclosure at the vertical wall is maintained at high temperature of T_h for three different cases of heater length (λ = 0.25, 0.5, 1). Fig.1 displays the enclosure with heater length (λ = 0.5) at the vertical wall. Inclined wall is kept cold and the bottom wall and the vertical wall excluding the heater is considered to be adiabatic. The cavity is filled with a non-Newtonian power-law fluid. The gravity is working in the negative direction of the coordinate system. For simplification no radiation effect has been taken into consideration.

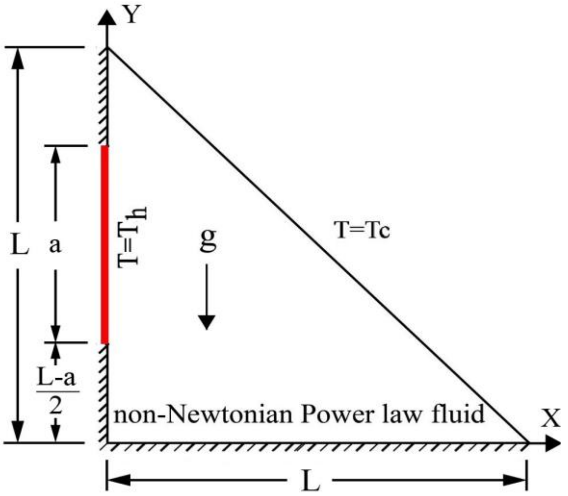

Fig.1: Schematic diagram of a triangular cavity filled with non-Newtonian fluid with discrete heater of $\lambda = a / L = 0.5$ at vertical wall.

3. MATHEMATICAL FORMULATION

Dimensional equations:

A set of governing equations has been formed assuming that a two-dimensional right angled triangular enclosure filled with an incompressible non-Newtonian power law fluid. The flow is incompressible and laminar. The density variation is approximated by the standard Boussinesq model. With invocation of Boussinesq's approximation, governing equations take the form as below:

The continuity equation-

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0. \qquad (1)$$

The momentum equations-

$$\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \frac{1}{\rho}\left(\frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}}\right) \qquad (2)$$

$$\rho\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{y}} + \left(\frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{yy}}{\partial \bar{y}}\right) + \rho g \beta (T - T_c) \qquad (3)$$

The energy equation-

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left[\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right] \qquad (4)$$

For a purely-viscous non-Newtonian fluid which follows the Ostwald–DeWaele (i.e. power-law) model the shear stress tensor is -

$$\tau_{ij} = 2\mu_a D_{ij} = \mu_a\left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i}\right) \qquad (5)$$

Here D_{ij} indicates the rate-of-deformation tensor for the two dimensional Cartesian coordinate and μ_a is the apparent viscosity that is derived for the two-dimensional Cartesian coordinates as

$$\bar{\mu}_a = K\left\{2\left[\left(\frac{\partial \bar{\mu}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\right] + \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)^2\right\}^{\frac{(n-1)}{2}} \qquad (6)$$

In the above equations $(\bar{u},\bar{v}), \bar{T}$ and $\bar{P}$ are the dimensional velocities, temperature and pressure respectively, ρ is the density and n is the power-law index. Therefore the deviation of *n* from unity indicates the degree of deviation from Newtonian behavior.

Non-dimensional equations:

In order to get the numerical solution of the system following scales are implemented to get the non-dimensional governing equations

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}, u = \frac{\bar{u}}{\left(\frac{\alpha}{L}\right)Ra^{0.5}}, v = \frac{\bar{v}}{\left(\frac{\alpha}{L}\right)Ra^{0.5}},$$

$$P = \frac{\bar{P}}{\rho\left(\frac{\alpha}{L}\right)^2 Ra}, \bar{T} = \frac{T - T_C}{T_H - T_C}. \qquad (7)$$

By substitution of Eq. (7) to Eqs. (1) - (4), the following system of equations is derived

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad (8)$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\Pr}{\sqrt{Ra}}\left[2\frac{\partial}{\partial x}\left(\frac{\mu_a}{K}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\mu_a}{K}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right)\right] \qquad (9)$$

$$\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \frac{\Pr}{\sqrt{Ra}}\left[2\frac{\partial}{\partial y}\left(\frac{\mu_a}{K}\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial x}\left(\frac{\mu_a}{K}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right)\right] \qquad (10)$$

$$u\frac{\partial \bar{T}}{\partial x} + v\frac{\partial \bar{T}}{\partial y} = \frac{1}{\sqrt{Ra}}\left[\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right] \qquad (11)$$

$$\mu_a = K\left\{2\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\}^{\frac{(n-1)}{2}} \qquad (12)$$

Boundary conditions used to solve the present problem are mentioned in Table 1.

Table 1. Boundary conditions in non- dimensional form.

Boundary wall	Flow field	Thermal field
Heater wall	U = 0,V = 0	$\overline{T} = 1$
Inclined wall	U = 0,V = 0	$\overline{T} = 0$
Bottom wall & Vertical wall excluding heater	U = 0,V = 0	$\frac{\partial \overline{T}}{\partial X} = 0$

The non-dimensional governing parameters used are Prandtl number (*Pr*) and Rayleigh number (*Ra*). These are defined below:

$$Pr = \frac{\nu_f}{\alpha_f}; Ra = \frac{g\beta_f (T_h - T_c)L^3}{\upsilon_f \alpha_f} \quad (13)$$

The characteristics of heat transfer are obtained by average Nusselt number and can be expressed as-

$$Nu_{av} = -\int_0^1 \frac{\partial \overline{T}}{\partial X} dY \quad (14)$$

4. NUMERICAL PROCEDURE

4.1 Grid Independency Test

An extensive mesh testing procedure is conducted to guarantee a grid independent solution and its results have been presented in Figure 2. To check the accuracy of the numerical solution several mesh element numbers (748, 1626, 2732 and 5234) have been checked. From the figure it is evident that velocity Y-gradient profiles almost overlap for element numbers 2732 and 5234. From the figure, it is seen that mid-plane Y-velocity profiles for grid elements of 2732 and 5234 almost overlap each other thus making the solution grid independent. So the optimum grid element for the present study is considered to be 5234 number of mesh elements and other numerical simulation has been carried out taking this grid as independent.

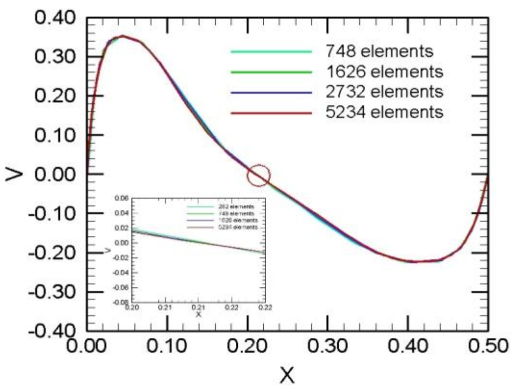

Fig.2: Variation of mid-plane Y-velocity with X-arc length at $\lambda = 1.0$, $Ra = 10^5$, $n = 0.5$ and $Pr = 10$.

4.2 Numerical Method

The Galerkin weighted residual finite element method (FEM) has been deployed to the present problem to obtain a numerical solution. From Boussinesq approximation, a set of algebraic equations has been formulated and the iterative process is used to solve this algebraic equation set. Triangular mesh formulation has been used to discretize the entire domain into several elements. Converging nature of the numerical solution has been confirmed and the converging criteria used is $|\Gamma^{n+1} - \Gamma^{n}| \leq 10^{-6}$ where *n* is the number of iteration and is general dependent variable.

4.3 Code Validation

Code validation test has been performed to compare the numerical accuracy of the present code with published studies in the literature on natural convection of non-Newtonian power-law in a cavity. Code validation is done through comparison of isotherm contour and streamlines with Kefayati et al. [18] at $Ra = 10^5$ and $Ha = 0$ in Fig. 3. So the present numerical code and solution procedure are completely reliable, and so is the numerical solution.

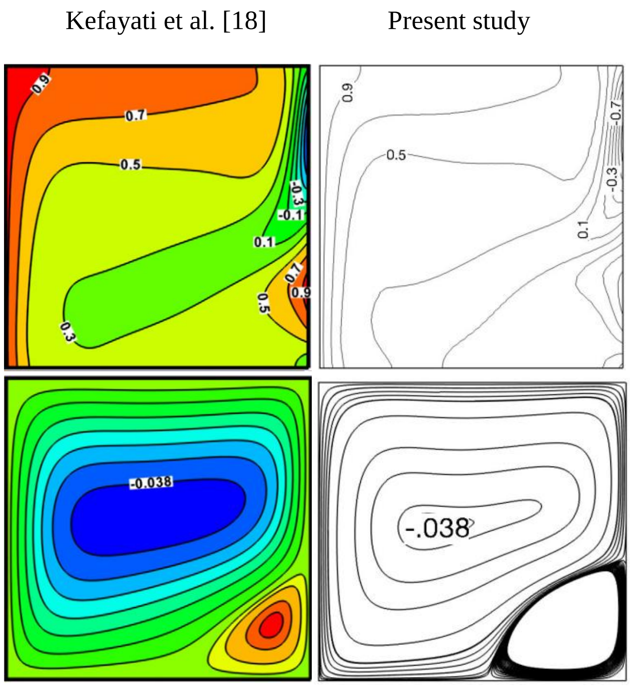

Fig.3: Comparison of isotherm and streamline contours between Kefayati et al. [18] and current case at $Ra = 10^5$ and $Ha = 0$.

5. RESULTS AND DISCUSSION

5.1 Effect of Power Law Index and Heater Length on Streamline and Isotherm:

Fig.4 (a) depicts the effect of power law index on streamline contours for different heater length at $Ra = 10^5$. For the specified problem with given boundary condition, the working fluid inside the triangular cavity absorbs heat from the heater wall and moves upward utilizing the buoyancy force. Again the fluid particle in contact with cold wall tends to fall downward following the traced path.

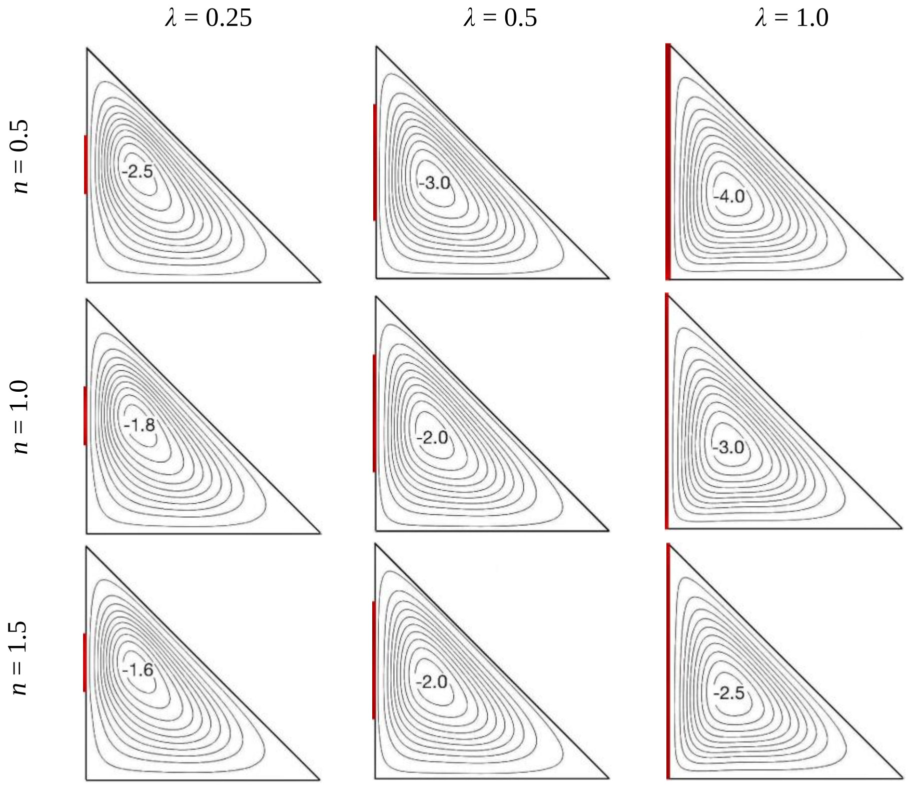

Fig.4 (a): Effect of power law index and heater size on streamline at $Ra = 10^5$ and $Pr = 10$.

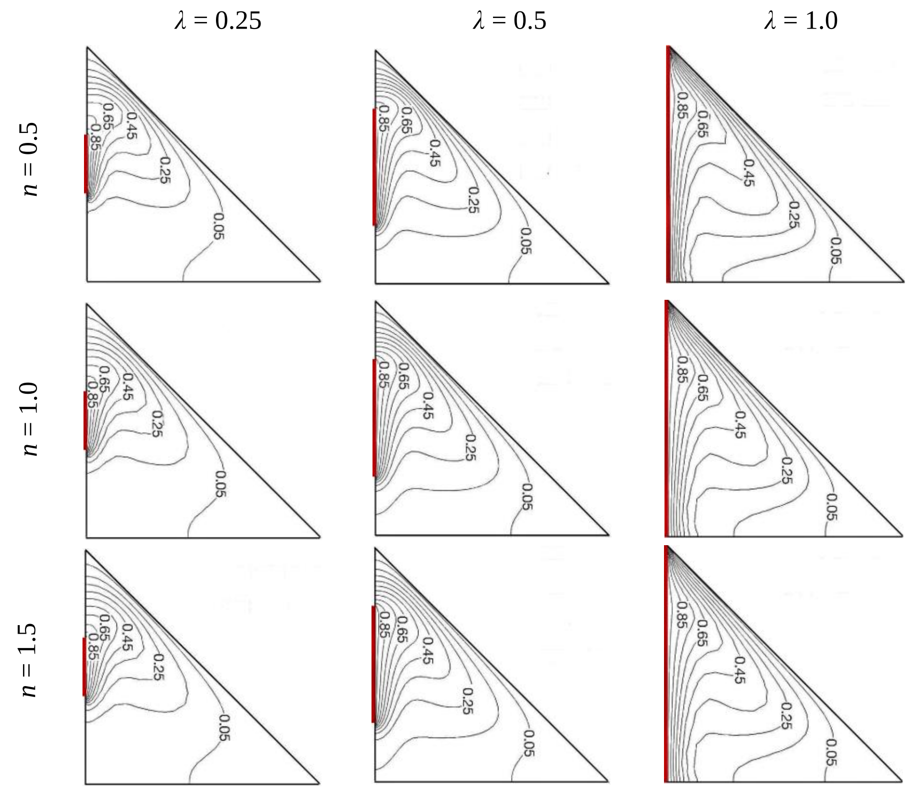

Fig.4 (b): Effect of power law index and heater size on isotherm at $Ra = 10^5$ and $Pr = 10$.

Such phenomena creates a counterclockwise vortex inside the cavity. As the power law index increases, the eddy created inside the cavity becomes weaker. Thus the fact is revealed that convection transport is stronger in pseudo-plastic fluids ($n < 1$) for heater length $\lambda = 1.0$ and weaker in dilatant fluid ($n > 1$) for $\lambda = 0.25$. This is due to the lower viscous characteristic of the pseudo-plastic fluids.

Fig. 4(b) establishes the thermal field of the fluid inside the cavity. More intensified and distorted contours are present for lower power law index, thus representing convective dominance. The convective dominance indicates a better heat transfer. As the fluid tends to act dilatant ($n = 1.5$), the patterns are diminished to be more parallel which is a consequence of conduction mode of heat transfer. Thus better convective thermal field is achieved at higher heater length ($\lambda = 1.0$) with pseudo-plastic fluid ($n = 0.5$) inside.

5.2 Effect of Power Law Index on Average Heat Transfer for Different Discrete Heater Length:

The overall heat transfer can be evaluated in terms of Nusselt number (*Nu*) which is illustrated in Fig. 5 for different heater length and power law index. Due to strong convective flow and higher thermal gradient, higher *Nu* is achieved for lower power law index. As the heater length increases, Nusselt number increases significantly and becomes maximum at $\lambda = 1.0$. This is obvious because $\lambda = 1.0$ ensures more heated surface area. As predicted in the thermal field discussion earlier, better heat transfer rate is obtained for pseudo-plastic fluid ($n = 0.5$) with heater length of $\lambda = 1.0$.

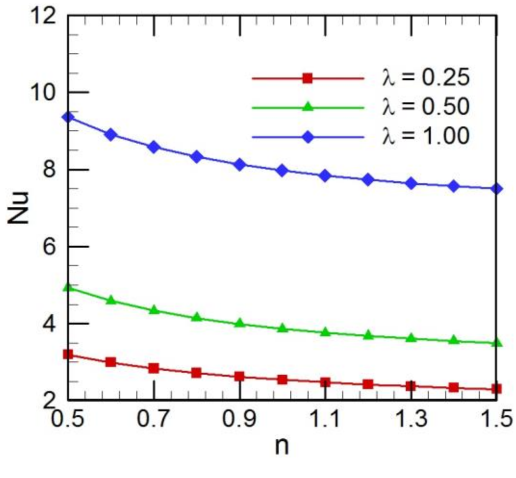

Fig.5: Variation of average Nusselt number with power law index for different discrete heater length at $Ra = 10^5$.

5.3 Effect of Rayleigh Number on Average Heat Transfer at Different Discrete Heater Length:

The variation of average Nusselt number with Rayleigh number (*Ra*) is illustrated in Fig.6 for different heater length. Pseudo-plastic fluid ($n = 0.5$) is chosen to investigate this very case as it ensures better heat transfer which is discussed earlier. As *Ra* increases, buoyancy force acting upon the working fluid increases

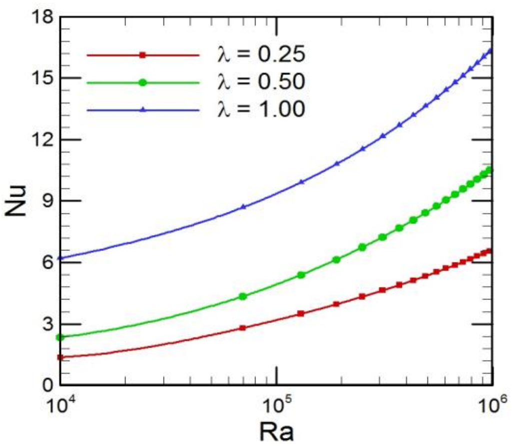

Fig.6: Variation of average Nusselt number due to the change of Rayleigh number at different discrete heater length at $n = 0.5$ and $Pr = 10$.

and more strong convection current is developed inside the cavity. This phenomenon results in increment of Nusselt number with the increment of Rayleigh number. Eventually the best heat transfer is achieved at $Ra = 10^6$ for maximum heater length of $\lambda = 1.0$.

6. CONCLUSION

Addressing to the effect of different pertinent parameters on heat transfer mechanism the following conclusive remarks can be drawn-

- Overall heat transfer is higher for pseudo-plastic fluid ($n = 0.5$) and lower for dilatant fluid ($n = 1.5$) than that for the Newtonian (n = 1) one.
- More than 42% better heat transfer is achieved for maximum heater length of $\lambda = 1.0$.
- Higher Rayleigh numbers ($Ra = 10^5$, 10^6) signify dominance of convection current while lower *Ra* ($Ra = 10^4$) ensures conduction regime.

To achieve better heat transfer the heater length should be high and working fluid must be of non-Newtonian pseudo-plastic characteristics.

7. ACKNOWLEDGEMENT

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9. NOMENCLATURE

Symbol	Meaning	Unit
a	heater length	(m)
g	acceleration due to gravity	$(m^{-}s^{2})$
k	thermal conductivity	$(W\ m^{-1}K^{-1})$
L	length of the triangular cavity	(m)
Nu	Average Nusselt number	Dimensionless
P	pressure	(Pa)
Pr	Prandtl number	Dimensionless
Ra	Rayleigh number	Dimensionless
T	temperature	(K)
u	velocity at x-direction	Dimensionless
v	dimensionless velocity at x-direction	Dimensionless
x	dimensionless distance along x-coordinate	Dimensionless
y	dimensionless distance along y-coordinate	Dimensionless
n	power law index	Dimensionless
	Greek symbols	
α	thermal diffusivity	(m^2/s)
β	volume expansion coefficient	(K^{-1})
μ	dynamic viscosity	(Nsm^{-2})
ν	kinematic viscosity	(m^2/s)
ρ	density	$(kg^{-}m^{3})$
ψ	stream function	
λ	Heater length	Dimensionless
	Subscripts	
h	hot	
c	cold	
f	base fluid (water)	