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Numerical solution of MHD free convection of heat and mass transfer flow over an inclined hot plate with viscous dissipation and Hall current

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Abstract*- The aim of the present work is an investigation of steady MHD free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a semi-infinite inclined hot plate under the influence of an applied uniform magnetic field with viscous dissipation & Hall current. The non-linear governing boundary layer equations together with the boundary conditions are transformed to a system of non-linear ordinary differential equations by using the similarity transformation. The system of non-linear ordinary differential equations are then solved by developing a suitable numerical techniques such as shooting iteration technique along with Runge- Kutta fourth order integration scheme. The numerical results concerned with the primary velocity, secondary velocity, temperature and concentration profiles effects of various parameters on the flow fields are investigated and presented graphically. The results presented graphically illustrate that primary velocity field rises due to increase of Hall parameter, Eckert number and angle of inclination parameter while secondary velocity increases for magnetic parameter and Hall parameter. Again the primary velocity field is decreased for magnetic parameter. The temperature field increases for the increasing values of magnetic parameter and Eckert number but up to a certain values of eta it is decreased and then increased for angle of inclination whereas reverse result arises for Prandtl number. Also, the negligible increasing effect on concentration profile are observed for increasing the values of magnetic parameter. Again, the concentration is decreased for Schmidt number and increased for inclination parameter and Eckert number. The numerical results for the heat flux from the stretching surface are compared with the results reported by other authors when the Grashof number, modified Grashof number, Hall current and inclination of the angle are absent. The present results in this paper are in good agreement with the work of the previous author.*

Keywords: MHD, Hall current, Viscous Dissipation

1. INTRODUCTION

The MHD flow problems has important in view of its significant applications in industrial manufacturing processes such as power generator, plasma studies, petroleum industries, cooling of Nuclear reactors, boundary layer control in aerodynamics etc. Also MHD laminar boundary layer flow over an inclined stretching sheet has noticeable applications in glass blowing, continuous casting, paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and spinning of fibbers. During its manufacturing process a stretched sheet interacts with the ambient fluid thermally and mechanically. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In the extrusion of a polymer sheet from a die, the sheet is some time stretched. By drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product of the desired characteristics can be achieved. In view of its significant application various authors has been done a lot of works related to this field such as , Venkatesulu and Rao [1] has considered the effect of Hall Currents and Thermo-diffusion on convective heat and mass ttransfer viscous flow through a porous medium past a vvertical porous plate, Sudha Mathew *et al*.[2] studied the Hall effects on heat and mass transfer through a porous medium in a rotating channel with radiation, Kumar and Singh [3] have studied Mathematical modeling of Soret and Hall effects on oscillatory MHD free convective flow of radiating fluid in a rotating vertical porous channel filled with porous medium, Chauhan and Rastogi [4] analyzed the effect of Hall current on MHD slip flow and heat transfer through a Porous medium over an accelerated plate in a rotating system and Nazmul Islam & Alam [5] studied

Dufour and Soret effects on steady MHD free convection and mass transfer fluid flow through a porous medium in a rotating system, Raptiset *et al*. [6] have studied the viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field. Tan *et al*. [7] studied various aspects of this problem, such as the heat, mass and momentum transfer in viscous flows with or without suction or blowing. Abel and Mahesh [8] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation. So the present paper is focused on steady MHD free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a semi-infinite inclined hot plate under the influence of an applied uniform magnetic field with viscous dissipation & Hall current.

2. FORMULATION OF THE PROBLEM AND SIMILARITY ANALYSIS

Let us consider steady two dimensional MHD free convection heat and mass transfer flow in an incompressible electrically conducting fluid over a semi-infinite inclined hot plate under the influence of an applied uniform magnetic field with viscous dissipation and Hall current. The flow is subjected to a transverse magnetic field of strength B_0 which is assumed to be applied in the positive y –direction normal to the surface. The pressure gradient, body force, viscous dissipation and joule heating effects are neglected. Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, energy and concentration under the influence of externally imposed magnetic field with the presence of Hall current are:

Equation of continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\alpha+ g\beta^*(C-C_\infty)\cos\alpha-\frac{\sigma B_0^2}{\rho(1+m^2)}(u+mW) \quad (2)$$

$$u\frac{\partial W}{\partial x}+v\frac{\partial W}{\partial y}=\nu\frac{\partial^2 W}{\partial y^2}+\frac{\sigma B_0^2}{\rho(1+m^2)}(mu-W) \quad (3)$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right]+\frac{\sigma B_0^2}{\rho c_p}\left(u^2+W^2\right) \quad (4)$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \quad (5)$$

where u and v are the velocity components along x and y directions and W is the secondary velocity component along z-axis, T, T_w and T_∞ are the fluid temperature, the stretching sheet temperature and the free stream temperature respectively while C, C_w and C_∞ are the corresponding concentrations, m is the hall parameter, k is the thermal conductivity of the fluid, c_p is the specific heat with constant pressure, α is the angle of inclination, μ is the coefficient of viscosity, ν is the kinematic viscosity, σ is the electrical conductivity, ρ is the fluid density, β is the thermal expansion coefficient, β* is the concentration expansion coefficient, B_0 is the magnetic field intensity, U_0 is the steam velocity, g is the acceleration due to gravity, D_m is the coefficient of mass diffusivity, respectively. The above equations are subject to the following boundary conditions:

$$u=U_0,\ v=0,\ W=0,\ T=T_w,\ C=C_w \quad \text{at} \quad y=0,$$
$$\text{and} \quad u=0,\ v=0,\ T=T_\infty,\ C=C_\infty \quad \text{as} \quad y\to\infty$$

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$W=U_0 g_0(\eta),\ \eta=y\sqrt{\frac{U_0}{2\nu x}},\ \psi=\sqrt{2\nu x U_0}\,f(\eta),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\ u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained

$$f'''+ff''+Gr\theta\cos\alpha+Gm\varphi\cos\alpha-\frac{M}{1+m^2}\left(f'+mg_0\right)=0 \quad (6)$$

$$g_0''+fg_0'+\frac{M}{1+m^2}\left(mf'-g_0\right)=0 \quad (7)$$

$$\theta''+\eta Prf\,\theta'+PrEc\left(f''^2+g_0'^2\right)+PrEcM\left(f'^2+g_0^2\right)=0 \quad (8)$$

$$\varphi''+Scf\varphi'=0 \quad (9)$$

The transform boundary conditions:

$$f=0,\ f'=1,\ g_0=0,\ \theta=\varphi=1 \text{ at } \eta=0$$
$$\text{and } f'=g_0=\theta=\varphi\to 0 \text{ as } \eta\to\infty$$

Where f', g_0, θ and φ are the dimensionless primary velocity, secondary velocity, temperature and concentration respectively, η is the similarity variable, η_∞ is the value of η at which boundary conditions is achieved, the prime denotes differentiation with respect to η. Also

$$M=\frac{2x\sigma B_0^2}{\rho U_0},\ Gr=\frac{2g\beta(T_w-T_\infty)x}{U_0^2},\ Gm=\frac{2xg\beta^*(C_w-C_\infty)}{U_0^2},$$

$$Pr=\frac{\mu c_p}{k},\ Sc=\frac{\nu}{D_m},\ \text{and}\ Ec=\frac{U_0^2}{c_p(T_w-T_\infty)}$$

are the magnetic parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number, and Eckert number respectively.

3. METHODOLOGY

The governing fundamental equations of momentum, thermal and concentration in Newtonian fluids are essentially nonlinear coupled ordinary or partial differential equations. Generally, the analytical solution of these nonlinear differential equations is almost difficult, so a numerical approach must be made. However no single numerical method is applicable to every nonlinear differential equation. The various types of methods that are available to solve these nonlinear differential equations are finite difference method, shooting methods, quasi-linearization, local similarity and non-similarity methods, finite element methods etc. Among these, the shooting method is an efficient and popular numerical scheme for the ordinary differential equations. This method has several desirable features that make it appropriate for the solution of all parabolic differential equations. Hence, the system of reduced nonlinear ordinary differential equations together with the boundary conditions have been solved numerically using fourth-order Runge-Kutta scheme with a shooting technique. Thus adopting this type of numerical technique described above, a computer program will be setup for the solution of the basic nonlinear differential equations of our problem where the integration technique will be adopted as the fourth order Runge-Kutta method along with shooting iterations technique. First of all, higher order non–linear differential equations are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem applying the shooting technique. Once the problem is reduced to initial value problem, then it is solved using Runge -Kutta fourth order technique. The effects of the flow parameters on the velocity, temperature and species concentration are computed, discussed and have been graphically represented in figures and also the values of skin friction, rate of temperature and rate of concentration shown in Table 1 for various values of different parameters. In this regard, defining new variables by the equations

$$y_1 = f,\ y_2 = f',\ y_3 = f'',\ y_4 = g_0,\ y_5 = g_0',$$
$$y_6 = \theta,\ y_7 = \theta',\ y_8 = \varphi,\ y_9 = \varphi'$$

The higher order differential equations (6), (7), (8) and (9) may be transformed to nine equivalent first order differential equations and boundary conditions respectively are given below:

$$dy_1 = y_2, dy_2 = y_3, dy_3 = -y_1 y_3 + \frac{M}{1+m^2} y_2$$
$$+\frac{Mm}{1+m^2} y_4 - Gr\cos\alpha\, y_6 - Gm\cos\alpha\, y_8, dy_4 = y_5,$$
$$dy_5 = -y_1 y_5 + \frac{M}{1+m^2} y_4 - \frac{Mm}{1+m^2} y_2, dy_6 = y_7,$$
$$dy_7 = -\eta Pr y_1 y_7 - PrEcy_3^2 - PrEcy_5^2 - PrEcM y_2^2$$
$$-PrEcM y_4^2, dy_8 = y_9, dy_9 = -Scy_1 y_9$$

The boundary conditions are as follows:

$$y_1 = 0, y_2 = 1, y_4 = 0, y_6 = y_8 = 1 \ \text{ at } \eta = 0,$$
$$\text{and } y_2 = y_4 = y_6 = y_8 \to 0 \ \text{ as } \ \eta \to \infty$$

4. RESULTS AND DISCUSSION

Numerical calculation for the distribution of primary velocity, secondary velocity, temperature and concentration profiles across the boundary layer for different values of the parameters are carried out. For the purpose of our simulation we have chosen M = 0.5, m = 0.3, Pr = 1.0, Gr = -2.0, Gm = -2.0, Ec = 0.2, Sc = 0.22, and $\alpha = 60^o$ while the parameters are varied over range as shown in the figures. The effects of various parameters on primary velocity profile are shown in Fig. 1- Fig.4. From Fig.1 it is observed that the primary velocity profile starts from maximum value at the surface and then decreasing until it reaches to the minimum value at the end of the boundary layer for all the values M. It is interesting to note that the effect of magnetic field is more prominent at the point of peak value, because the presence of M in an electrically conducting fluid introduces a force like Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in the present problem. As a result velocity profile is decreased. Again, from Fig.2 and Fig.3 it is observed that the velocity is increased for the increasing values of Hall parameter and angle of inclination. Besides, Fig.4 shows the negligible increasing effect of Eckert number on primary velocity profile. Fig. 5 - Fig.8 show the secondary velocity profile for various values of M, m, α and *Ec*. From these figures it is observed that the secondary velocity profile is increased for the increasing values of M and m but decreasing effect are observed in case of Eckert number whereas up to a certain values of eta the secondary velocity profile is decreased and then increased for increasing values of inclination parameter. Again, Fig. 9 – Fig. 12 show the temperature profile obtained by the numerical simulation for various values of entering parameters. From Fig.9 and Fig.11 it is clearly demonstrates that the thermal boundary layer thickness increases for the increasing values of magnetic parameter and Eckert number because heat energy is stored in the liquid due to the frictional heating. It is obvious that the increasing effect of Ec enhanced the temperature at any point of flow region. From this figure it is concluded that greater viscous dissipative heat be the cause of increasing temperature profile. From Fig. 10 it is observed that in the certain interval of η, the temperature profile is decreased and then increased for increasing values of inclination parameter but reverse trend arises for the increasing values of Pr which shown in Fig.12. That is, the thermal boundary layer thickness increases as the Pr increases implying lower heat transfer. It is due to the fact that grater values of Pr means increasing thermal conductivity and therefore it is able to diffuse heat from the plate more slowly than smaller values of Pr, hence the rate of heat transfer is reduced as a result the heat of the fluid in the boundary layer increases. Again Fig.13 - Fig. 16 shows the concentration profiles obtained by the numerical simulation for various values of entering non-dimensional parameters. From Fig.15 it is seen that the concentration profile is increased for the effect of Ec due to the fact that heat energy is stored in the liquid due to the frictional heating but reverse results arises for the increasing values of magnetic parameter which is shown in Fig.13. It is obvious that the increasing effect of Ec enhanced the concentration at any point of flow region for the case of present work. From this figure it is concluded that greater viscous dissipative heat be the cause of increasing concentration profile. Again, the concentration profile is increased for the increasing values of inclination parameter whereas reverse results arises in case of Schmidt number are shown in Fig.14 and Fig.16. Again a negligible increasing effect are observed in case of magnetic parameter which are shown in Fig.13.Further the numerical solutions for the skin friction and the Nuselt number have been compared with those of Ishak *et al*. [9].These results are given in Table.1 and it is observed that the present results and those of Ishak *et al*. [9] are in good agreement.

5. CONCLUSIONS

In the present paper is an investigation of steady MHD free convection, heat and mass transfer flow of an incompressible electrically conducting fluid over a stretching sheet in a rotating system under the influence of an applied uniform magnetic field with Hall current. The leading equations are solved numerically by the shooting method along with Runge- Kutta fourth-fifth order integration scheme. The results are presented to display the flow characteristic like velocity, temperature and concentration. Following are the conclusions made from above analysis:

- The magnitude of primary velocity profile decreases with increasing magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity but there is no effect on primary velocity profile for rotational and reaction parameter.
- The secondary velocity starts from minimum value at the plate and increases until it attains the maximum value within the boundary layer and then starts decreasing until it reaches the free stream area satisfying the far field boundary condition. Therefore the noticeable increasing effect are observed for magnetic and Hall parameter on secondary velocity profile but reverse result arises for Eckert number. It is interesting to note that up to a certain values of eta the secondary velocity profile is decreased and then increased for increasing values of inclination parameter.
- The temperature profile is starting at the initial point of the plate surface and increases until it attains the maximum value within the boundary layer and then starts decreasing until it reaches to zero far away from the plate satisfying the boundary condition. So noticeable increasing effect are observed on temperature profile for magnetic parameter and Eckert number. Again up to a certain interval of η, the temperature profile is decreased and then increased for increasing values of inclination parameter but reverse trend arises for the increasing values of Pr. That is, the thermal boundary layer thickness increases as the Pr increases implying lower heat transfer. It is due to the fact that grater values of Pr means increasing thermal conductivity and therefore it is able to diffuse heat from the plate more slowly than smaller values of Pr, hence the rate of heat transfer is reduced as a result the heat of the fluid in the boundary layer increases In this case temperature asymptotically approaches to zero in free stream region.
- The effect of Sc is found to decrease the concentration profile because increasing in Sc decreases molecular diffusivity which result a decrease of the boundary layer. Hence the concentration of the species in lower for large values of Sc. Again, negligible increasing effect are observed for the remaining parameters.

Table 1.Comparison of skin friction [$f''(0)$] and rate of heat transfer [$-\theta'(0)$] for different values of *M*, when *Pr = 1.0, Gr = 0, Gm = 0, Ec = 1.0, Sc = 0.22, α = 0*, m = 0.

M	Ishak et al. [9]		Present results	
	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.0	0.5607	1.0873	**0.5627**	**1.0891**
0.1	0.5658	1.0863	**0.5718**	**1.0882**
0.2	0.5810	1.0833	**0.5985**	**1.0851**
0.5	0.6830	1.0630	**0.6910**	**1.0663**
1.00	1.0000	1.0000	**0.9985**	**0.9987**
2.00	1.8968	0.8311	**1.9015**	**0.8254**
5.00	4.9155	0.4702	**4.9244**	**0.4806**

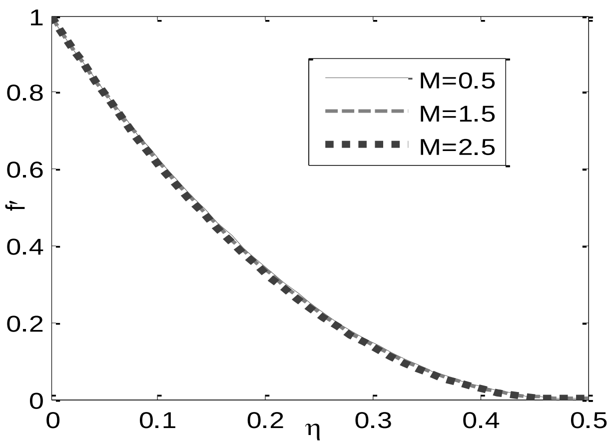

Fig.1: Primary velocity profile for various values of M

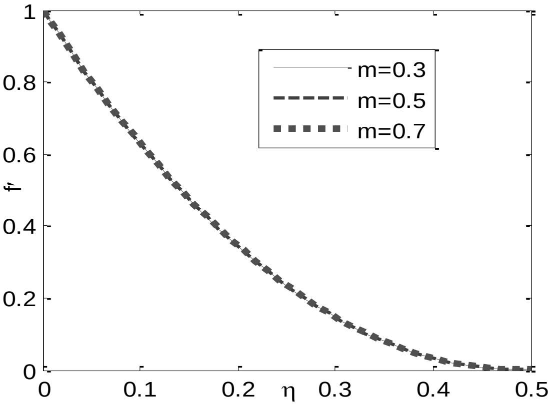

Fig.2: Primary velocity profile for various values of m

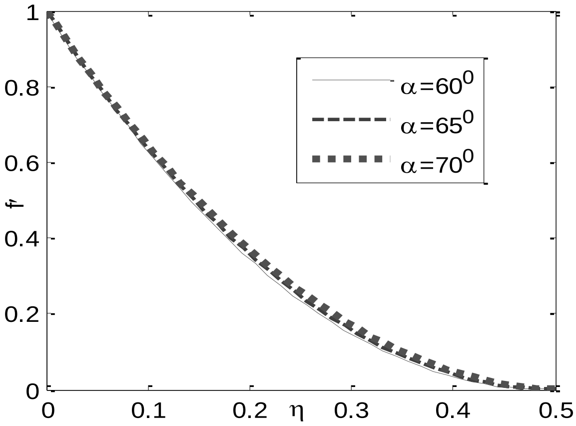

Fig.3: Primary velocity profile for various values of α

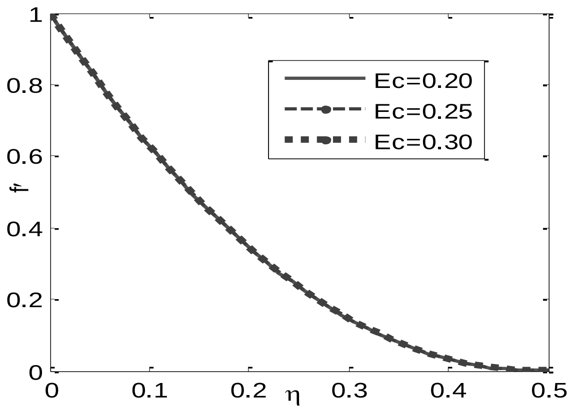

Fig.4: Primary velocity profile for various values of Ec

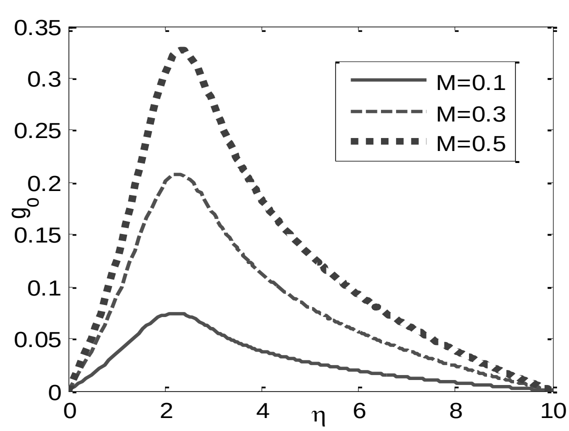

Fig.5: Secondary velocity profile for various values of M

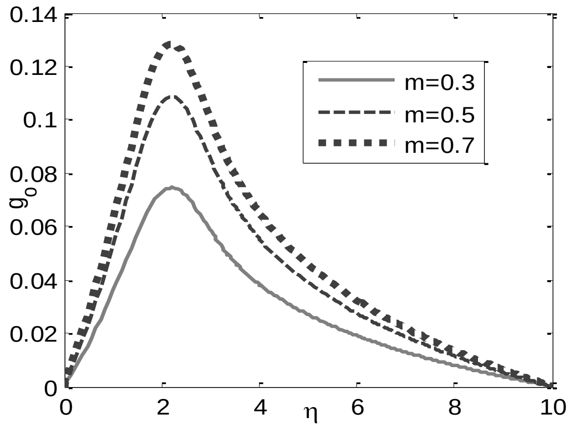

Fig.6: Secondary velocity profile for various values of m

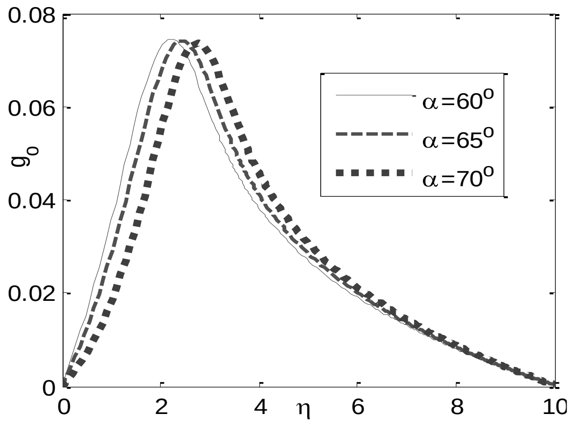

Fig.7: Secondary velocity profile for various values of α

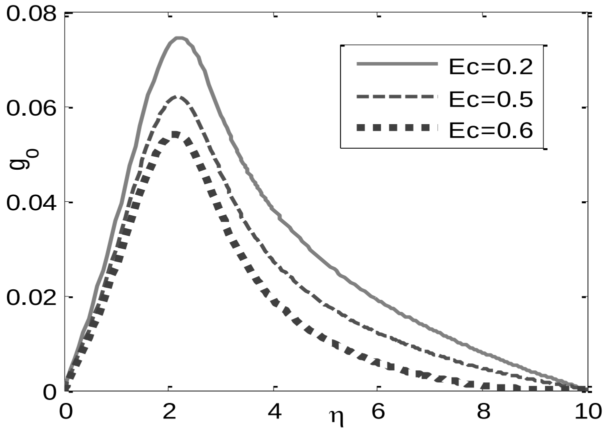

Fig.8: Secondary velocity profile for various values of Ec

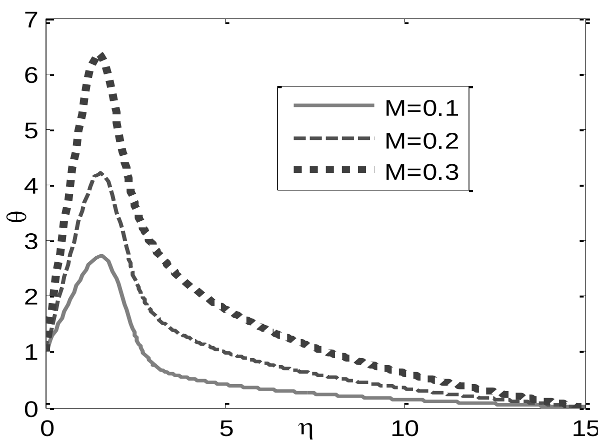

Fig. 9: Temperature profile for various values of M

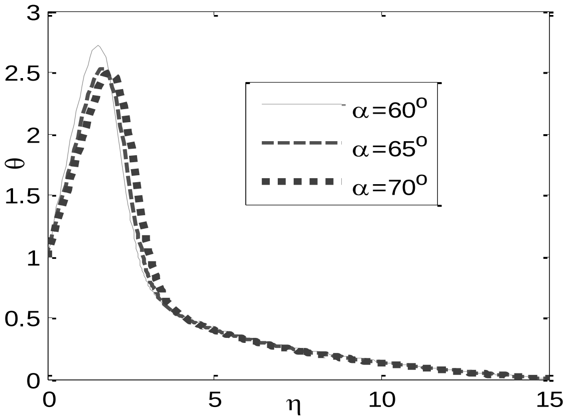

Fig. 10: Temperature profile for various values of α

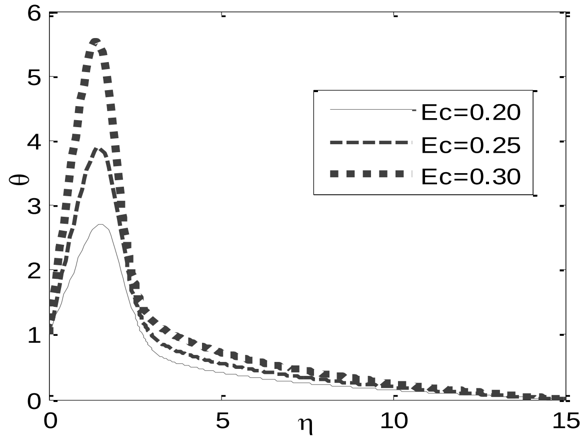

Fig. 11: Temperature profile for various values of Ec

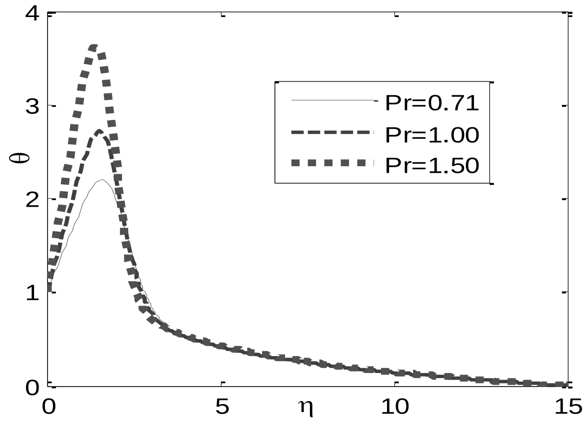

Fig. 12: Temperature profile for various values of Pr

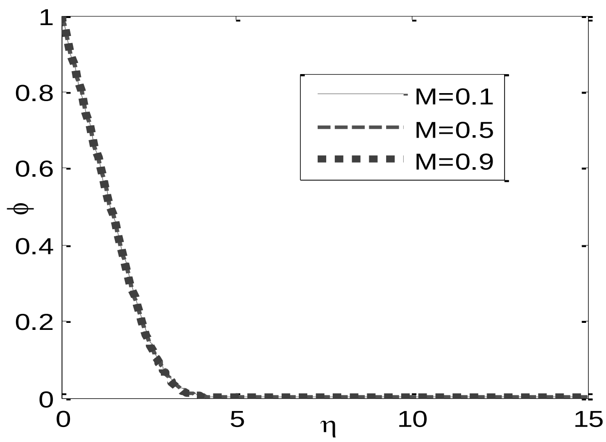

Fig. 13: Concentration profile for various values of M

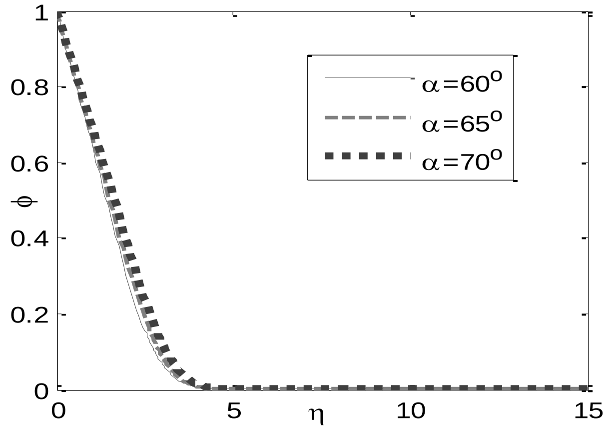

Fig. 14: Concentration profile for various values of α

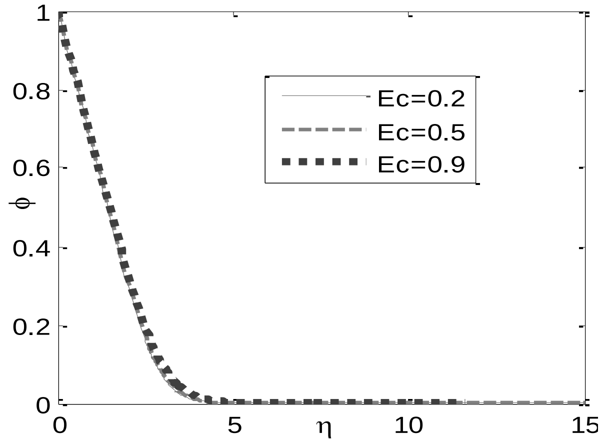

Fig. 15: Concentration profile for various values of Ec

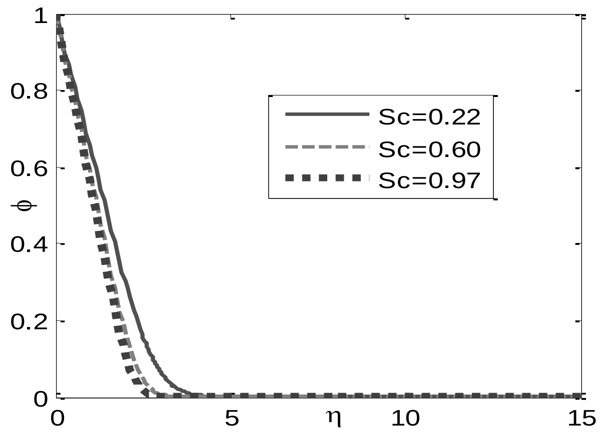

Fig. 16: Concentration profile for various values of Sc

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