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BOUNDARY LAYER ANALYSIS ALONG A STRETCHING WEDGE SURFACE WITH MAGNETIC FIELD IN A NANOFLUID

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Abstract- The present problem performs the boundary layer flow, heat and mass transfer in a nanofluid along a stretching wedge with the effect of magnetic field. The equation is formulated in terms of momentum, energy and nanoparticle concentration along with boundary conditions. The mathematical formulation is based on Falkner – Skan boundary layer equation and Buongiorno model. The governing partial differential equations are transformed into ordinary differential equations by applying local similarity transformations and solve them numerically. The results of velocity, temperature and nanoparticle concentration profiles are displayed in graphically and also the local skin friction coefficient, rate of heat and mass transfer are in tabular form.

Keywords: MHD, Hall current, Viscous Dissipation

1. INTRODUCTION

MHD laminar boundary layer flow over an inclined stretching sheet has noticeable applications in glass blowing, continuous casting, paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and spinning of fibbers. During its manufacturing process a stretched sheet interacts with the ambient fluid thermally and mechanically. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In the extrusion of a polymer sheet from a die, the sheet is some time stretched. By drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product of the desired characteristics can be achieved. In view of its significant application various authors has been done a lot of works related to this field such as , Venkatesulu and Rao [1] has considered the effect of Hall Currents and Thermo-diffusion on convective heat and mass ttransfer viscous flow through a porous medium past a vertical porous plate, Sudha Mathew *et al*.[2] studied the Hall effects on heat and mass transfer through a porous medium in a rotating channel with radiation, Kumar and Singh [3] have studied Mathematical modeling of Soret and Hall effects on oscillatory MHD free convective flow of radiating fluid in a rotating vertical porous channel filled with porous medium, Chauhan and Rastogi [4] analyzed the effect of Hall current on MHD slip flow and heat transfer through a Porous medium over an accelerated plate in a rotating system and Nazmul Islam & Alam [5] studied Dufour and Soret effects on steady MHD free convection and mass transfer fluid flow through a porous medium in a rotating system, Raptiset *et al*. [6] have studied the viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field. Tan *et al*. [7] studied various aspects of this problem, such as the heat, mass and momentum transfer in viscous flows with or without suction or blowing. Abel and Mahesh [8] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation. So the present paper is focused on steady MHD free convection, heat and mass transfer nanofluid flow of an incompressible electrically conducting fluid along a stretching wedge shape surface.

2. GOVERNING EQUATIONS OF THE PROBLEM

AND SIMILARITY ANALYSIS

Let us consider steady two dimensional MHD laminar boundary layer flow of an incompressible, electrically conducting, viscous Newtonian fluid past a stretching wedge surface which is electrically non-conducting semi-infinite sheet with heat and mass transfer. The stretching sheet is permeable to allow for possible blowing or suction, and is continuously stretching in the direction of x-axis. The flow is along the wedge surface which is measured the x-axis and y-axis is perpendicular to it. Two equal and opposite forces are applied along the x-axis so that the wall is stretched with a velocity $u = u_w(x) = ax^m$ and keeping the origin fixed. The surface temperature T_w and nanoparticle concentration C_w are maintained at non-uniform temperature which is greater than the free stream temperature T_∞ and nanoparticle concentration C_∞ . The uniform transverse magnetic field B_o is imposed parallel to the y-axis and the induced magnetic field due to the motion of the electrically conducting fluid is negligible since for small magnetic Reynolds number. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. The total angle of the wedge is $\Omega = \beta\pi$. The velocity of the wedge surface is $u_w(x)$, the free stream velocity is $U(x)$, the temperature of the wedge is T_w and nanoparticle concentration C_w are respectively defined as follows $u_w(x) = ax^m, U(x) = bx^m, T_w = T_\infty + bx^m,\ C_w = C_\infty + bx^m$

Where a and b are positive constant and the exponent m (pressure gradient parameter) is a function of the wedge angle parameter β where the total apex angle of the wedge is βπ such that

$$m = \frac{\beta}{2-\beta} \quad \text{or} \quad \beta = \frac{2m}{1+m}\ .$$

Therefore, the governing partial differential equations of continuity, momentum, energy and nanoparticle concentration are as follows
presence of Hall current are:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu_f \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0{}^2}{\rho_f}(U-u) \qquad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_f \frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \qquad (3)$$

Nanoparticle concentration equation:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The above equations are subject to the following boundary conditions:

$$u = u_w(x),\ v = 0,\ T = T_w, C = C_w \text{ at } y = 0 \text{ and}$$
$$u = U(x), T \to T_\infty, C \to C_\infty \text{ as } y \to \infty$$

where u and v are the velocity components along x and y directions, ν_f is the kinematic viscosity of the base fluid, ρ_f is the density of the base fluid, σ is the electrical conductivity, B_0 is the magnetic field intensity, g is the acceleration due to gravity, α_f is the thermal diffusivity of the base fluid, DB is the Brownian diffusion coefficient, DT is the thermophoresis diffusion coefficient. Here τ is the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid, T is the fluid temperature and C is the nanoparticle concentration respectively.

To convert the governing equations into a set of ordinary differential equations, we introduce the following similarity transformations:

$$\eta = y\sqrt{\frac{U(1+m)}{2x\nu}},\ \psi = \sqrt{\frac{2x\nu U}{(1+m)}}\, f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, u = \frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\psi}{\partial x}$$

By applying the above similarity transformations, the partial differential Eq. (2) – Eq.(4) are transformed into non-dimensional, nonlinear and coupled ordinary differential equations as follows:

$$f''' + ff'' + \beta(1 - f'^2) + \lambda(M + K^*)(1+m)^{-1}(1 - f') = 0 \qquad (5)$$

$$\theta'' + Pr\left[Nb\,\theta'\varphi' + Nt\,\theta'^2 + f\,\theta' - \beta\, f'\theta\right] = 0 \qquad (6)$$

$$\varphi'' + \frac{Nt}{Nb}\theta'' + Le\, Pr\left[f\,\varphi' - \beta\, f'\varphi\right] = 0 \qquad (7)$$

The transformed boundary conditions:

$$f = 0\ f' = \lambda, \theta = \varphi = 1, f = 1 \text{ at } \eta = 0,$$
$$\text{and } f' \to 1, \theta = \varphi \to 0 \text{ as } \eta \to \infty$$

Where

$$M = \frac{2\sigma B_0^2 x^{1-m}}{\rho_f a}, \lambda = \frac{a}{b}, \ Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu_f},$$

$$Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu_f}, \beta = \frac{2m}{1+m}, Pr = \frac{\nu_f}{\alpha_f},$$

$$Le = \frac{\alpha}{D_B}, K^* = \frac{2\nu x^{1-m}}{Ka}$$

are the magnetic parameter, velocity ratio parameter, Brownian motion parameter, thermophoresis parameter, pressure gradient parameter, Prandtl number,Lewis number and porosity parameter respectively. The important physical quantities of this problem are skin friction coefficient C_f, the local Nusselt number Nu and the local Sherwood number Sh which are proportional to rate of velocity, rate of temperature and rate of nanoparticle concentration respectively.

3. METHODOLOGY

The governing fundamental equations of momentum, thermal and concentration in Newtonian fluids are essentially nonlinear coupled ordinary or partial differential equations. Generally, the analytical solution of these nonlinear differential equations is almost difficult, so a numerical approach must be made. However no single numerical method is applicable to every nonlinear differential equation. The various types of methods that are available to solve these nonlinear differential equations are finite difference method, shooting methods, quasi-linearization, local similarity and non-similarity methods, finite element methods etc. Among these, the shooting method is an efficient and popular numerical scheme for the ordinary differential equations. This method has several desirable features that make it appropriate for the solution of all parabolic differential equations. Hence, the system of reduced nonlinear ordinary differential equations together with the boundary conditions have been solved numerically using fourth-order Runge-Kutta scheme with a shooting technique. Thus adopting this type of numerical technique described above, a computer program will be setup for the solution of the basic nonlinear differential equations of our problem where the integration technique will be adopted as the fourth order Runge-Kutta method along with shooting iterations technique. First of all, higher order non–linear differential equations are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem applying the shooting technique. Once the problem is reduced to initial value problem, then it is solved using Runge -Kutta fourth order technique. The effects of the flow parameters on the velocity, temperature and species concentration are computed, discussed and have been graphically represented in figures and also the values of skin friction, rate of temperature and rate of concentration shown in Table 1 for various values of different parameters. In this regard, defining new variables by the equations

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta y_5 = \theta', y_6 = \varphi, y_7 = \varphi'$$

The higher order differential equations (5), (6), and (7) may be transformed to seven equivalent first order differential equations and boundary conditions respectively are given below:

$$y_1' = y_2, y_2' = y_3$$
$$y_3' = -\lambda\left(M + K^*\right)(1+m)^{-1}(1 - y_2) - y_1 y_3 - \beta\left(1 - y_2^2\right)$$
$$y_4' = y_5, y_5' = -Pr\left[Nby_5 y_7 + Nty_5^2 + y_1 y_5 - \beta y_2 y_4\right]$$
$$y_6' = y_7, y_7' = -\frac{Nt}{Nb} y_5'^2 - LePr\left[y_1 y_7 - \beta y_2 y_6\right]$$

The transformed boundary conditions:

$$y_1(0) = 0, y_2(0) = \lambda, y_3(0) = \alpha_1, y_4(0) = 1, y_5(0) = \alpha_2,$$
$$y_6(0) = 1, y_7(0) = \alpha_3 \text{ at } \eta = 0$$
$$y_2(\infty) \to 1, y_4(\infty) \to 0, y_6(\infty) \to 0 \text{ as } \eta \to \infty$$

Where the unknowns α_1, α_2 and α_3 are determined such that $y_2(\infty) \to 1, y_4(\infty) \to 0, y_6(\infty) \to 0$ as $\eta \to \infty$. The essence of this method is that first the boundary value problem is converted to an initial value problem and then use a shooting numerical technique to guess the values of α_1, α_2 and α_3 until the boundary conditions $y_2(\infty) \to 1, y_4(\infty) \to 0, y_6(\infty) \to 0$ as $\eta \to \infty$ are satisfied. The resulting differential equations are then easily integrated using fourth order classical Runge-Kutta method.

4. RESULTS AND DISCUSSION

Numerical solution are obtained using the above numerical scheme for the distribution of velocity, temperature and nanoparticle concentration profiles across the boundary layer for different values of the parameters. The velocity profiles for various dimensionless parameters have been shown in Fig. 1 – Fig. 4. From these figures it is observed that the velocity profiles increases for increasing values of magnetic parameter, pressure gradient parameter and porosity parameter as a result the boundary layer thickness decreases but the reverse result arises in case of stretching ratio parameter. Figure 5 – Figure 7 shows the temperature profiles for various entering parameters. From these figures it is observed that the heat transfer rate increases for increasing values of Prandtl number and Brownian motion as a result the thermal boundary layer thickness decreases but reverse trend arises for thermophoresis parameter. The nanoparticle concentration have been shown in Fig.8 – Fig. 11. The concentration decreases for increasing values of thermophoresis parameter, stretching ratio and Lewis number but increases for Brownian motion parameter. Also, the numerical values of skin friction coefficient, rate of heat transfer and rate of mass transfer has been shown in Table 1.

5. CONCLUSIONS

From the above analysis the main observation is the velocity profile is exist up to $\beta > -0.35$ but in Falker – Skan problem it was $\beta > -0.198$. The pressure gradient parameter, thermophoresis parameter and stretching ratio parameter is the key factor to enhance heat and mass transfer rate.

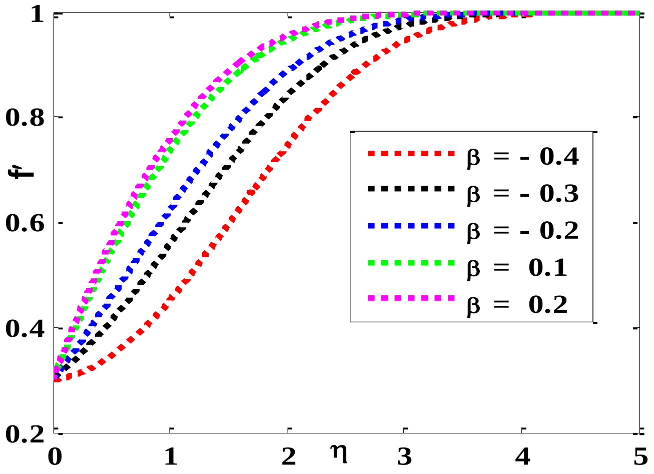

Fig.1: Velocity profile for various values of β

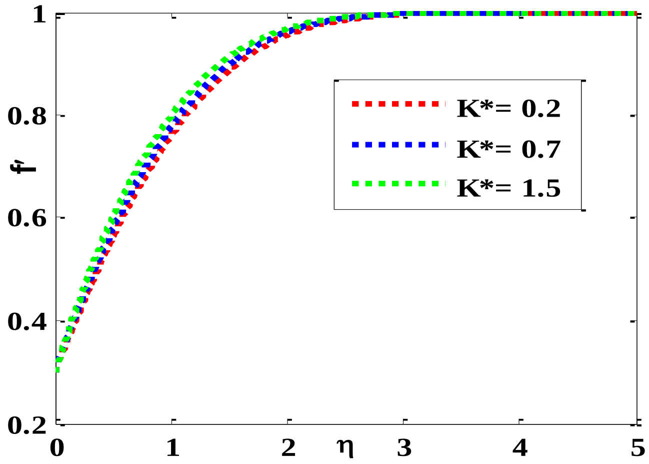

Fig.2: Velocity profile for various values of K*

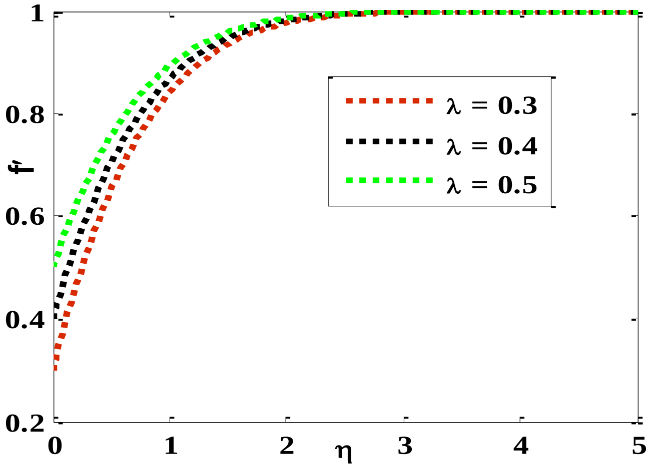

Fig.3: Velocity profile for various values of λ

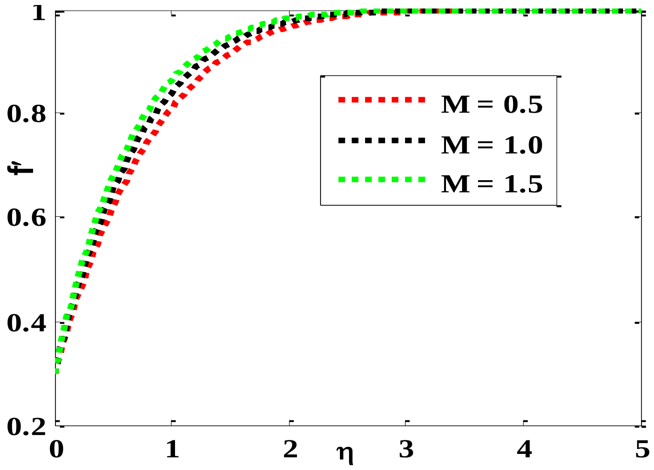

Fig.4: Velocity profile for various values of M

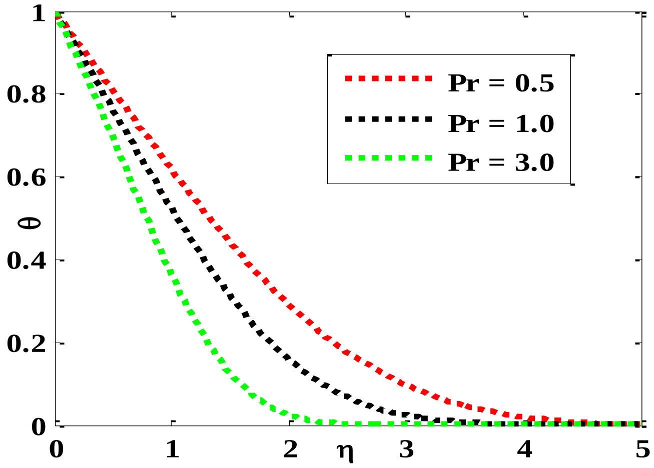

Fig.5: Temperature profile for various values of Pr

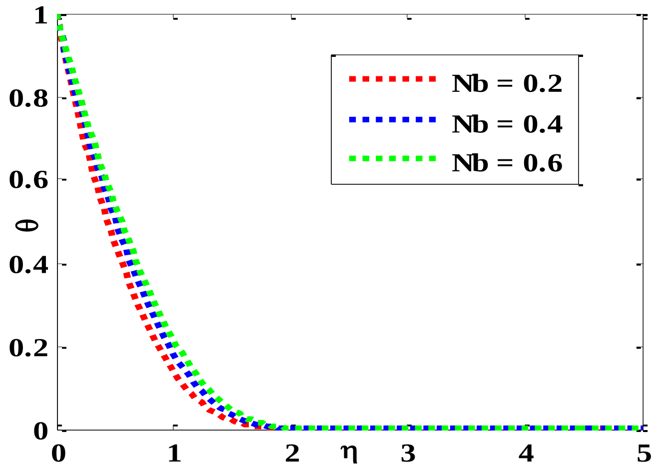

Fig.6: Temperature profile for various values of Nb

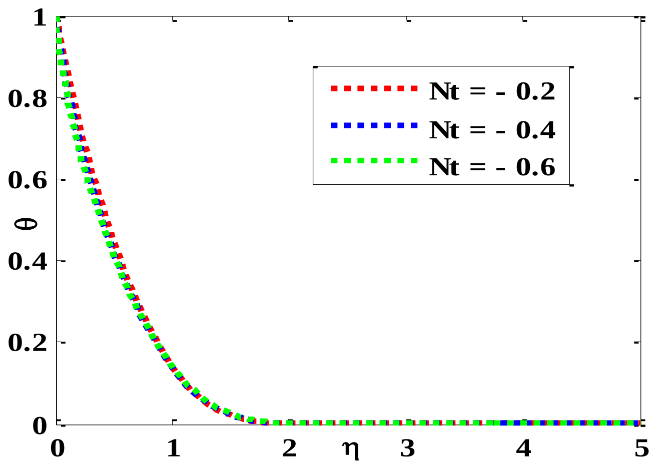

Fig.7: Temperature profile for various values of Nt

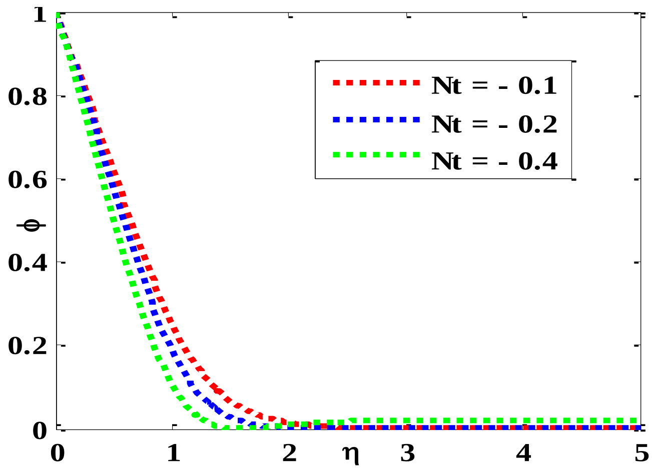

Fig.8: Nanoparticle concentration profile for Nt

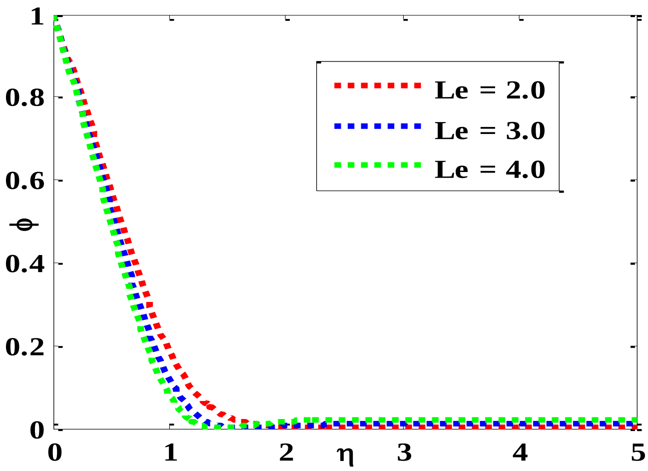

Fig. 9: Nanoparticle concentration profile for Le

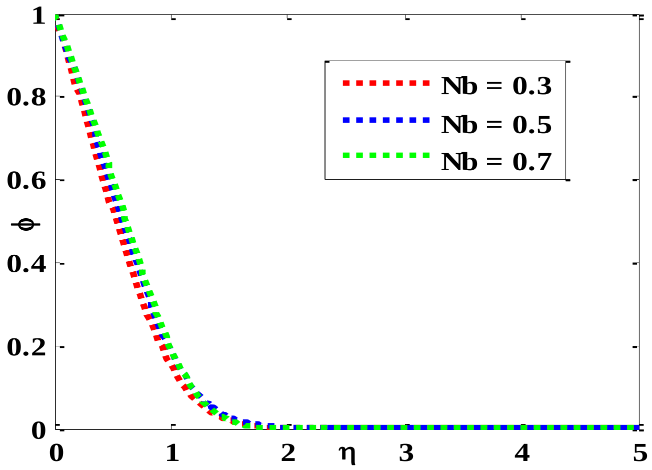

Fig. 10: Nanoparticle concentration profile for Nb

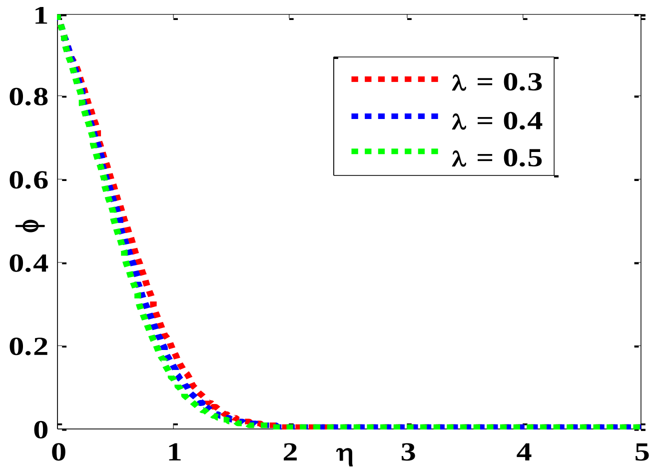

Fig. 11: Nanoparticle concentration profile for λ

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8. NOMENCLATURE

Symbol	**Meaning**	**Unit**
MHD	Megnetohydrodynamic	(K)
α_f	Thermal diffusivity of the base fluid	mm^2s^{-1}
K	Thermal conductivity	w $m^{-1}K^{-1}$
σ	Electrical conductivity	sm^{-1}
D_B	Brownian diffusion coefficient	-
D_T	Thermophoresis diffusion coefficient	-
ν_f	kinematics viscosity of the base fluid,	m^2s^{-1}
ρ_f	density of base fluid,	kg m^{-3}
B_0	Magnetic field intensity,	Am^{-1}
Ψ	stream function	
u	Velocity component along x -axis	ms^{-1}
v	Velocity component along y -axis	ms^{-1}
a	Stream velocity constant	
b	Free stream velocity constant	
τ	ratio of the effective heat capacity	
λ	stretching ratio	
C	nanoparticle volume fraction	kg m^{-3}
C_w	plate volume fraction	kg m^{-3}
C_∞	free stream nanoparticle volume fraction	
T	fluid temperature	k^{-1}
T_w	plate temperature	k^{-1}
T_∞	free stream temperature	