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# NUMERICAL STUDY OF NATURAL CONVECTION HEAT TRANSFER IN PARTIALLY HEATED SQUARE ENCLOSURE FILLED WITH NANOFLUID

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***Abstract***- *This study investigates natural convection heat transfer of water-based nanofluid in a square enclosure, where the left wall is maintained at constant higher temperature and right wall is maintained at lower temperature and other walls remain adiabatic. Water based nanofluid contains copper as nanoparticle, which is considered to be in unsteady state. The influence of significant parameters such as volume fraction of nanoparticle, Rayleigh number has been studied. The range of volume fraction (φ) and Rayleigh number (Ra) is taken [0, 0.1], and [$5\times10^4$, $5\times10^6$], respectively. DNS simulation has been performed with Ansys fluent 16.1(student version). The results show that the local and average heat transfer rate increases significantly as particle volume fraction increases. However, Nusselt number (both local and average) decreases with increase in volume fraction. Substantial increment in local and average heat transfer rate and Nusselt number is observed with increase in Rayleigh number*.

**Keywords:** Convection, Square enclosure, Nanofluid, Volume fraction, DNS.

## 1. INTRODUCTION

Buoyancy induced flow and heat transfer is an important phenomenon in engineering systems due to its wide applications in thermal insulation, heating and cooling of buildings, energy drying processes, lakes and geothermal reservoirs, solar collector etc. [1]. Enhancement of heat transfer performance in these systems is an essential topic from an energy saving perspective. Fluids in common use, such as water, oil and ethylene glycol, often have low thermal conductivity of conventional heat transfer, a primary limitation in enhancing the performance and the compactness of many electronic devices for engineering applications. One innovative way to improve the thermal conductivity of a fluid is to suspend metallic Nano particles within it. The resulting mixture, referred to as a nano fluid, which is firstly utilized by Choi [2]. There is currently a lack of sophisticated theories for predicting the effective thermal conductivity of a nanofluid, several researchers have proposed different correlations to predict the apparent thermal conductivity of two-phase mixtures. The models proposed by Hamilton and Crosser [3], Wasp [4], Maxwell-Garnett [5], Bruggeman [6] and Wang *et al*. [7] are all meant to determine the effective thermal conductivity of a nanofluid, but all have failed to predict it accurately. To be specific, experimental results have shown much higher thermal conductivities than those predicted by these models. An alternative expression for calculating the effective thermal conductivity of solid–liquid mixtures was proposed by Yu and Choi [8]. The past decade has witnessed several studies of convective heat transfer in nanofluids. Khanafer *et al*. [9] were the first to investigate the problem of buoyancy-driven heat transfer enhancement of nanofluids in a two-dimensional enclosure. Putra *et al* [10] did the same. Jou and Tzeng [11] numerically investigated the heat transfer performance of nanofluids inside two dimensional rectangular enclosures. Their results show that increasing the volume fraction causes a significant increase in the average heat transfer coefficient. But there is a contradiction in case of Nusselt number’s response with volume fraction. Santra *et al*. [12] have conducted a similar kind of study, up to φ = 10%, using the models proposed by Maxwell-Garnett and Bruggeman. Their results showed that the Bruggemann model predicts higher heat transfer rates than the Maxwell-Garnett model. Hwang *et al*. [13] have carried out a theoretical investigation of the thermal characteristics of natural convection of an alumina-based nanofluid in a rectangular cavity heated from below using Jang and Choi’s model [14] for predicting the effective thermal conductivity of nanofluids (and various models for predicting the effective viscosity). Oztop and Abu-Nada [15] investigated heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using nanofluids using various types of nanoparticles A numerical study on natural convection in a glass-melting tank heated locally from below has been performed by Sarris *et al*.[16]. More recently Calcagni *et al*. [17] made an experimental and numerical study of free convective heat transfer in a square enclosure characterized by a discrete heater located on the lower wall and cooling from the lateral walls.

In this paper effect of volume fraction of nanoparticle in heat transfer in a square enclosure is observed. Effect of Rayleigh number in heat transfer in a square enclosure filled with nanofluid also investigated.

## 2. PHYSICAL MODEL

Figure 1 shows a schematic diagram of the partially heated square enclosure. The fluid in the enclosure is a water based nanofluid containing Cu as nanoparticles. The nanofluid is assumed to be in unsteady state. The flow is assumed to be laminar. It is assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo-physical properties of the nanofluid are given in Table 1. The left wall is hot wall, maintained at a constant temperature $\mathbf{T_h}$. The right wall is cold wall, maintained at temperature $\mathbf{T_c}$**.** Temperature of left wall is higher than the right wall. Top and bottom walls are insulated. All four walls remain stationary. Here we assume that fluid velocity at all fluid–solid boundaries is equal to that of the solid boundary i.e. no slip condition and fluid temperature at all fluid–solid boundaries is equal to that of the solid boundary wall temperature i.e. no jump condition. The thermo-physical properties of the nanofluid are assumed to be constant except for the density variation, which is approximated by the Boussinesq model.

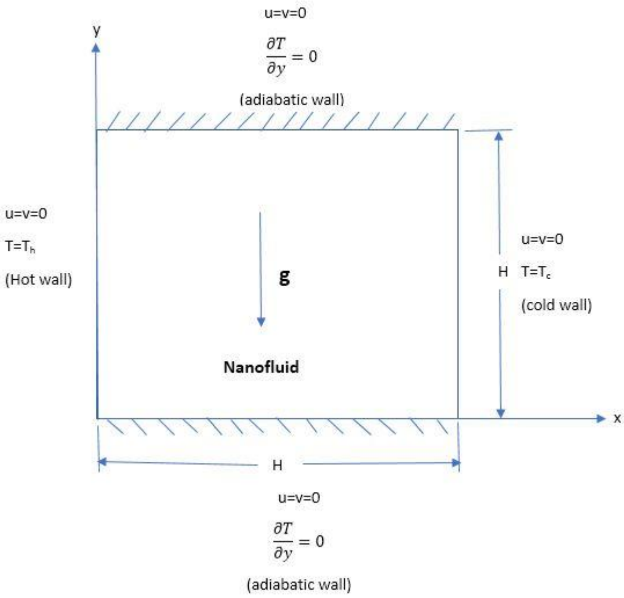



Fig.1: A typical sketch which shows the Problem with necessary condition.

## 3. COMPUTATIONAL DETAILS

The general momentum equation is also called the equation of motion or the Navier-Stoke's equation; in addition, the equation of continuity is frequently used in conjunction with the momentum equation. The equation of continuity is developed simply by applying the law of conservation of mass to a small volume element within a flowing fluid. The governing continuity, momentum and energy equations are-

Continuity: $$\frac{\partial \rho_{nf}}{\partial t} + \frac{\partial}{\partial x}(\rho_{nf}\, u) + \frac{\partial}{\partial y}(\rho_{nf}\, v) = 0 \qquad (1)$$

x momentum: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial P}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

y momentum: $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial P}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho_{nf}}(\rho\beta)_{nf}\, g(T - T_c) \qquad (3)$$

Energy: $$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q'}{\rho C_p} \qquad (4)$$

Here the term -ρg on the right side of the eq. (3) represents the body force exerted on the fluid element in the negative y direction [18].

The Boundary conditions are

- u (x, 0) =0, v(x,0) = 0 ;$0 < x < H$
- u (x, H) =0, v (x, H) = 0 ;$0 < x < H$
- u (0, y) =0, v (0, y) = 0 ;$0 < y < H$
- u (H, y) =0, v (H, y) = 0 ;$0 < y < H$
- T (x, 0) = $T_h$ = 310 K
- T (x, H) = $T_c$ = 300 K
- $\frac{\partial T}{\partial y} = \mathbf{0}$ at T (x, 0) and T (x, H).

The following dimensionless parameters are defined to show different results later:

$$X = \frac{x}{W} \qquad Y = \frac{y}{H} \qquad \theta = \frac{T - T_c}{T_h - T_c} \qquad (16)$$

The effective properties of nanofluid are calculated as follows-

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s \qquad (17)$$

$$\alpha_{nf} = \frac{k_{eff}}{(\rho c_p)_{nf}} \qquad (18)$$

The heat capacitance of the nanofluid is expressed as (Abu-Nada [19]; Khanafer *et al.* [9]):

$$(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_s \qquad (19)$$

The effective thermal conductivity of the nanofluid is approximated by the Maxwell–Garnetts [5] model

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \qquad (20)$$

The use of eq. (20) is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles. The viscosity of the nanofluid can be approximated as viscosity of a base fluid lf containing dilute suspension of fine spherical particles and is given by Brinkman [20]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \qquad (21)$$

Physical property of the Cu-water nanofluid is determined following Table 1.

Table 1: Thermo-physical properties of base fluid and Nano-particles [21]

| Property | | Water | Cu |
|---|---|---|---|
| $\rho$ | **(kg/m$^3$)** | 997.1 | 8933 |
| $C_p$ | **(J/kg.K)** | 4179 | 385 |
| $K$ | **(W/m.K)** | 0.613 | 400 |
| $\alpha$***10$^7$** | **(m$^2$/s)** | 1.47 | 1163.1 |
| $\beta$ | **(K$^{-1}$)** | 0.00021 | 0.000051 |

## 4. RESULT AND DISCUSSION

### 4.1 Model Validation

The model is investigated by using Ansys Fluent 16.1(student version). It has been validated against solutions obtained in the literature as shown in Table 2. Natural convection of air inside a square cavity whose two sides are set to differential temperatures while keeping the top and bottom surfaces at adiabatic condition is a classic case for validation. Average Nusselt number for the hot wall calculated from the present model is compared with the data available in the literature and found very accurate for various Rayleigh numbers.

Table 2: Comparison of average Nusselt number for the hot wall obtained by various studies with present study for different Rayleigh numbers.

| **Ra** | **Nu** | | | | | | |
|---|---|---|---|---|---|---|---|
| | Present work | Vahl Davis [22] | Fusegi *et al.*[23] | Comini *et al.* [24] | Khanafer *et al.* [9] | Bilgen [25] | Kobra [26] |
| $10^4$ | 2.240668 | 2.243 | 2.302 | | 2.245 | 2.245 | 2.2448 |
| $10^5$ | 4.526406 | 4.519 | 4.646 | 4.503 | 4.522 | 4.521 | 4.5216 |
| $10^6$ | 8.862388 | 8.799 | 9.012 | 8.825 | 8.826 | 8.8 | 8.8262 |

### 4.2 Effect of Volume Fraction of Nanoparticle on Heat Transfer

Contours of static temperature, stream function are shown in Fig.2. Contours are drawn for the time of 120 sec. For same time, with increasing volume fraction there is significant difference in the contours. For lower volume fraction change in temperature distribution is less than that of higher volume fraction. For high volume fraction change in temperature distribution in left upper side and right lower side is more prominent than lower volume fraction. Similarly, in case of stream function (Fig.3), maximum value of stream function increases with increase in volume fraction and contour takes a regular shape in higher volume fraction. The change is due to equivalent thermal conductivity.

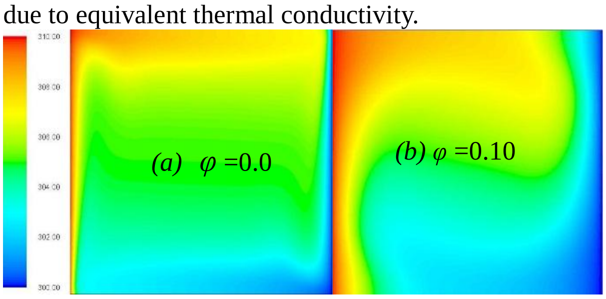



Fig.2: Contours of Static temperature with and without Nano particle (Ra=5×10$^6$, time=120 sec).

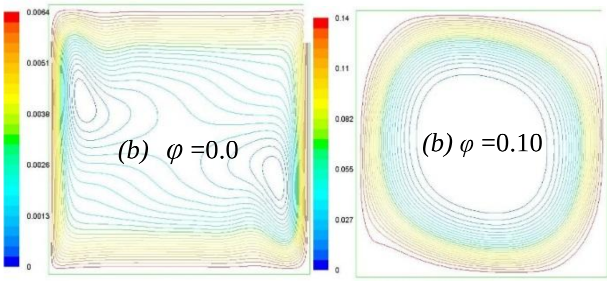
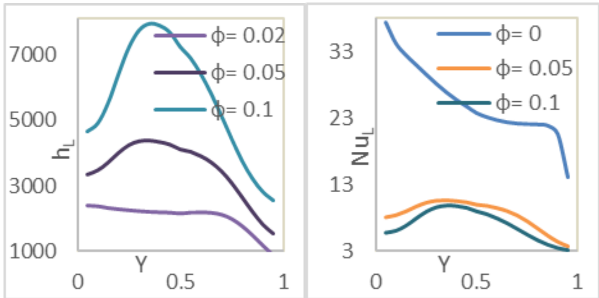


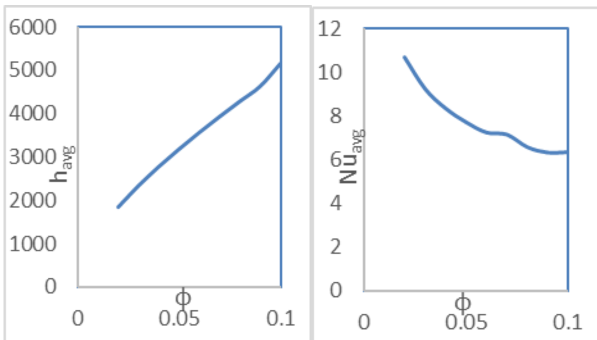

Fig.3: Contours of stream function(streamlines) with and without Nano particles (Ra=5×106, time=120 sec).

Figure 4 shows variation of local heat transfer coefficient and nusselt number at hot wall and Fig.5 shows average heat transfer coefficient and nusselt number at hot wall for different volume fraction. Figure 4 shows that local heat transfer coefficient increases with increase in volume fraction whereas local nusselt number decreases with increases in volume fraction. Considering single graph in Fig.4 it is seen that heat transfer coefficient first increases to a maximum value, then starts to decrease and goes to a minimum value. Fig.5 shows that average heat transfer coefficient increases linearly with increase in volume fraction and nusselt number decreases with increase in volume fraction. Which justify the previous studies [9,10].




(a) (b)

Fig.4: Variation of local (a) convective heat transfer coefficient and (b) nusselt number along hot wall for different volume faction (Ra=5×10$^6$, time=120 sec)




(a) (b)

Fig.5: Variation of average (a) convective heat transfer coefficient and (b) nusselt number with volume fraction along hot wall (Ra=5×10$^6$, time=120 sec).

### 4.3 Effect of Rayleigh Number on Heat Transfer

Contours of static temperature, stream function are shown in Fig.6. Contours are drawn for the time of 120 sec. For same time, with increasing Rayleigh number there is significant difference in the contours. For smaller value of Rayleigh number there is slight change in the contour, but with increment of Rayleigh number there is a visible change in temperature. In case of stream function (Fig.7), with increase of Rayleigh number maximum magnitude of stream function also increases. These phenomena are due to buoyancy effect.

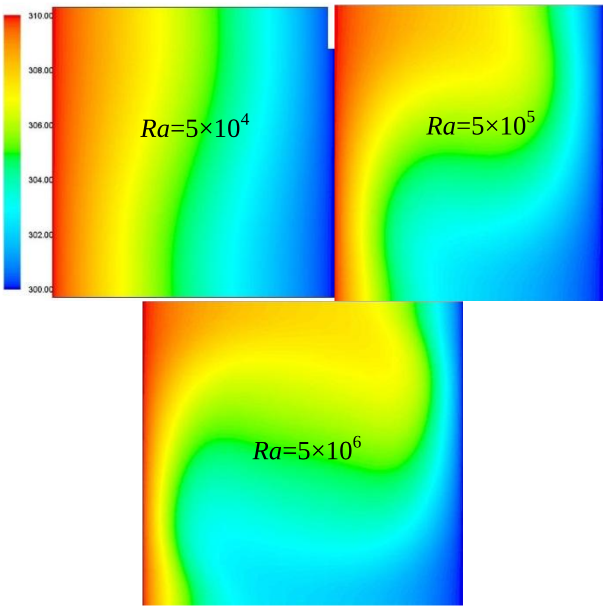



Fig.6: Contours of Static temperature for different Rayleigh number ($\varphi$ = 0.10, time=120 sec).

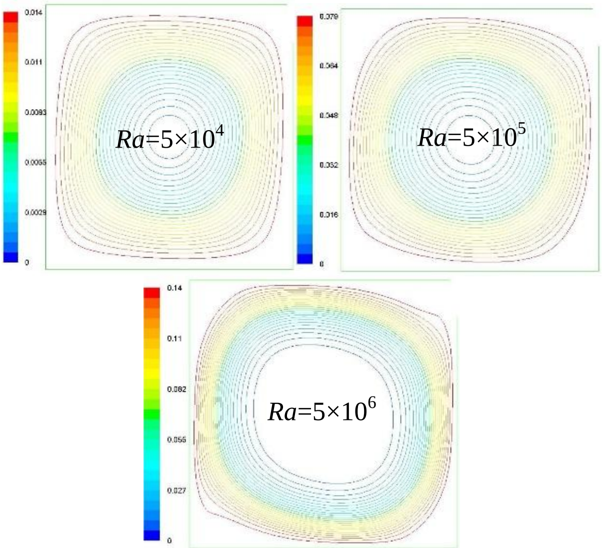
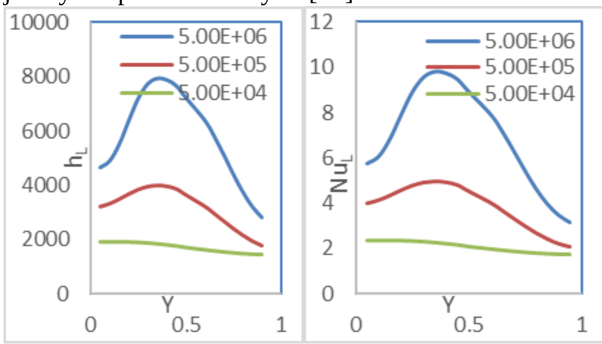


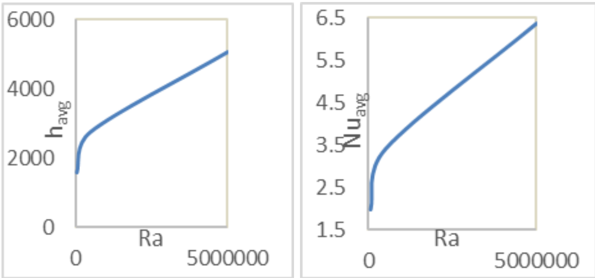

Fig.7: Contours of Stream function (streamline) for different Rayleigh number ($\varphi$ = 0.10, time=120 sec).

Local heat transfer coefficient and nusselt number increases with increase in Rayleigh number, as shown in Fig.8. For both case of heat transfer coefficient and nusselt number, considering a single curve, it is seen that its value first increases to a peak value then starts to decrease. As shown in Figure 9 it is clear that, both the average heat transfer coefficient and nusselt number increases with increase in Rayleigh number. Which justify the previous analysis [15].




*(a)* *(b)*

Fig.8: Variation of local (a) convective heat transfer coefficient and (b) nusselt number along hot wall ($\varphi$ = 0.10, time=120 sec).




*(a)* *(b)*

Fig.9: Variation of average (a) convective heat transfer coefficient and (b) Nusselt number with Rayleigh number ($\varphi$ = 0.10, time=120 sec).

## 5. CONCLUSION

A comprehensive investigation on natural convection in a square enclosure filled with Nano fluid is presented. The investigation is done to show the enhancement in heat transfer due to use of Nano fluid instead of using pure fluids. The parameters investigated are, the solid volume fraction, aspect ratio and the Rayleigh number. The result clearly shows that-

- ❖ The amount of heat transfer is increased remarkably with increase in volume fraction of nanoparticles, but the nusselt number decreases with increase in the same.
- ❖ Substantial increment in heat transfer and nusselt number is observed with increase in Rayleigh number.

## 6. FUTURE WORKS RECOMMENDATIONS

In future, there are lots to do. The study can be extended by doing followings-

- ❖ Using different types of nanoparticles such as Ag, $TiO_2$, CuO, $Al_2O_3$ , to determine which nanoparticle is more effective in enhancement in heat transfer.
- ❖ In this analysis volume fraction range was [0,10]. This range can be extended to observe the effect of higher volume fraction.

- ❖ Diameter of droplet is factor, which can influence the heat transfer rate. So, variation of droplet diameter to show its effect on heat transfer can be a nice topic.
- ❖ Heat transfer can be investigated for higher range of Rayleigh number. Different turbulence model can be used to analyze heat transfer for high Rayleigh number ranging to turbulent flow.
- ❖ This investigation involves partial heating from left wall. This can be varied. There is various combination of partially heated enclosure. Several combinations give new possibility in heat transfer rate enhancement.

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## 8. NOMENCLATURE

| Symbol | Meaning | Unit |
|---|---|---|
| $H$ | Height & Width of enclosure | (m) |
| $\varphi$ | Volume fraction | |
| $T$ | Temperature | (K) |
| $P$ | Pressure | (Pa) |
| $t$ | Time | (sec) |
| $x, y$ | Dimensional space | (m) |

| | | |
|---|---|---|
| | coordinate | |
| $g$ | Gravitational acceleration | $(m/s^{-2})$ |
| $Ra$ | Rayleigh no | |
| $h$ | Heat transfer coefficient | $(W/m^{-2}K^{-1})$ |
| $\alpha$ | Thermal Diffusivity | $(m^2s^{-1})$ |
| $\beta$ | Thermal expansion coefficient | $(K^{-1})$ |
| $\mu$ | Dynamic viscosity | $Nsm^{-2}$ |
| $\upsilon$ | Kinematic viscosity | $(m^2s^{-1})$ |
| $\rho$ | Density | $(kg/m^3)$ |
| $k$ | Thermal conductivity | $(W/m^{-1}K^{-1})$ |
| $C_p$ | Specific heat | $(Jkg^{-1}K^{-1})$ |
| $c$ | Cold | |
| $h$ | Hot | |
| $eff$ | Effective | |
| $h_{avg}$ | Average convective heat transfer coefficeint | $(W/m^{-2}K^{-1})$ |
| $Nu_{avg}$ | Average Nusselt number | |
| $h_L$ | Local convective heat transfer coefficient | $(W/m^{-2}K^{-1})$ |
| $Nu_L$ | Local Nusselt number | |
| $s$ | Solid | |
| $f$ | Base fluid | |