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WEIGHTED COST BASED DISTRIBUTION OPPORTUNITY TABLE IN TRANSPORTATION PROBLEM

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Abstract*- Transportation Problems (TP) has been playing very important role to ensure in time availability of raw materials and finished goods from different sources to distinct destinations. Only a strong network based on a suitable transportation algorithm can minimize the transportation cost and time. Many researchers have developed a numbers of transportation algorithms and also research works are ongoing for better results. Much of the research works concern with cost matrix, we mean, manipulation of cost entries to form Distribution Indicator (DI) or Total Opportunity Cost (TOC) table whatever be the structure of supply row and demand column. It is observed that none of the works consider* ***Supply*** *and/or* ***Demand*** *for formulation of DI table or TOC. But frequently it is observed that supply and demand play a vital role for the formulation of cost allocation table to obtain a better solution. In this article a weighted cost based Distribution opportunity table is formulated by considering supply and demand entries as a weight factor. Experiments have been carried out to justify the validity and the effectiveness of the proposed weighted based total opportunity cost table.*

Keywords: Transportation Problems, Distribution Indicator, Total Opportunity Table, Supply and Demand.

1. INTRODUCTION

Transportation Problems (TP) play a very important role to ensure in time availability of raw materials and finished goods from different sources to distinct destinations. Only a strong network based on a suitable transportation algorithm can minimize the transportation cost and time. Now-a-days, communication lines, railroad networks, pipeline systems, road networks, shipping lines, aviation lines etc. are typical examples of network. In all these networks, we are interested to send some specific commodity from certain supply places to some demand places. Many researchers have developed a numbers of transportation algorithms and also research works are ongoing for better results. Moreover for finding Initial Basic Feasible Solution (IBFS) much of the research works are concerned with cost matrix and manipulation of cost matrix. It is noted that in TP, all the optimized algorithms initially need an IBFS to obtain the optimal solution.

There are various simple heuristic methods available to get an IBFS, such as, North-West Corner method, Row minimum method, Column minima Method, Least Cost Matrix method etc. [1]. Among all the simple heuristic methods, the Least Cost Matrix (Matrix Minima) is relatively efficient and this method considers the lowest cost cell of the Transportation Table (TT) for making allocation in every stage. There is another well-known algorithm for IBFS is "VAM—Vogel's Approximation Method"[2]. After VAM method, researchers proposed several versions of the VAM method by modifying some tricks [3- 11]. Recently, [12] presented an alternative method to North West Corner (NWC) method by using Statistical tool called Coefficient of Range (CoR). It is noted that, all the approaches discussed above are concerned with the cost entries and /or the manipulation of cost entries to form DI or TOC table whatever be the structure of supply and demand. None of them considered to treatment in cost elements by manipulating supply/ demand to find DI or TOC in allocation procedures. But it might be assumed that, supply and demand play a vital role in the formulation of cost allocation table to obtain a better solution.

2. MATHEMATICAL MODEL OF BALANCED TRANSPORTATION PROBLEM

2.1 Mathematical model of balanced transportation problem

Before formulation of weighted distribution table based Transportation Problem (TP), it is worthwhile to present a mathematical model of a general balanced TP. In order to minimize the transportation costs, the general formulation of the transportation problem is as follows:

$$\text{Minimize} \quad \mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i\, , i = 1,2,\cdots,m \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = b_i\, , i = 1,2,\cdots,n \tag{3}$$

$$x_{ij} \geq 0 \; \forall \; i, j \tag{4}$$

The distributions of unit cost to transport from origin O_i to destination D_j which is denoted as c_{ij} as well as demand and supply can be shown in a tabular form given as follows:

Table 1. Tabular view of a Transportation Problem (TP)

	Destinations						
Origins		D_1	D_2	⋯	⋯	D_n	Supply
	O_1	c_{11}	c_{12}	⋯	⋯	c_{1n}	a_1
	O_1	c_{21}	c_{22}	⋯	⋯	c_{2n}	a_2
	⋮	⋮	⋮	⋯	⋯		
	⋮	⋮	⋮	⋯	⋯		
	O_m	c_{m1}	c_{m2}	⋯	⋯	c_{mn}	a_m
	Demand	b_1	b_2	⋯	⋯	b_n	
	Requirement						

2.2 Formulation of Weighted Distribution Cost Table

It is a role of thumb that maximum supply will be done where transportation cost is minimum. Upon this idea researcher developed the well-known Least Cost Matrix method to find optimal solution in TP. But reality is that, most of the time it provides the IBFS which is not good enough. It is also a general practice in business arena that among the several demands shopkeeper want to sell where demand is maximum so that he can able to sell maximum but saving several sell parameters such as time, manpower etc. Therefore amount of supply and demand could play a vital role in business arena. Exploit this idea a weighted cost based distribution table will be formulated by considering supply and demand entries as a weight factor.

Finding cell weight: At first, it will be tried to identity the valid weight factor of each cell cost by considering supply and demand entries. It is noted that the maximum possible allocation of the cell C_{ij} is $min\ (S_i, D_j)$, where S_i denotes total supply at node i and D_j indicates total demand at node j. If the cost matrix consists of m sources and n destinations, then cost matrix contains mn cost cells. Now as the maximum ability of allocation of each cell C_{ij} is $min\ (S_i, D_j)$, so the total possible maximum allocation of all cells be $\sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j)$. Therefore for each cell C_{ij}, its cell's weight factor is :

$$min\ (S_i, D_j) / \sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j)$$

Apply weight factor to each cell: Now as in natural role of sense, smaller cost cell has larger priority for allocation of goods. So it is needed to formulate cell costs so that, smaller cell cost provided larger opportunity to have larger weight factor. By exploiting this concept, we have formed a virtual weighted cost at cell C_{ij}, as :

$${}^{w}c_{ij} = \frac{1}{c_{ij}} \times \ min\ (S_i, D_j) / \sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j) \tag{5}$$

where ${}^{w}c_{ij}$ and c_{ij} denote weighted virtual cell cost and actual value of cost at the cell C_{ij}, respectively provided each cell cost $c_{ij} \neq 0$. In the case of $c_{ij} = 0$

$${}^{w}c_{ij} = M \times min\ (Si, Dj) / \sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j) \tag{6}$$

where M is defined as follow:

(a) If $\{c_{ij}: 0 < c_{ij} < 1 \; \forall i, j \;\} \neq \Phi$ (i.e. null set) then set $M = max\{S_i\ , D_j\ \forall i, j\} / [min\ \{c_{ij}: 0 < c_{ij} < 1\ \forall i, j\ \}]$.

(b) Else set $M = max\ \{S_i\ , D_j\ \forall i, j\}$

Therefore in this way a virtual Weighted Cost Distribution Table (WCDT) will be formulated.

Allocation procedure: Allocation procedure is very simple, similar to the Least Cost Matrix approach but here we consider weighted virtual cost rather than exact cost. That is, allocate to the cell which corresponds to ***maximum virtual weighted cost*** rather than ***minimum cost***. So if there is more than one identical cell cost among the all identical cell costs we have obviously obtained a larger virtual weighted cost in one of those cells if $min\ (S_i, D_j)$ are not identical.

3. EXPERIMENTATION AND DISCUSSION

For the justification and effectiveness of the proposed allocation procedure, we consider a typical example given in Table 2.

Table 2. A typical example of TP

	D_1	D_2	D_3	D_4	Supply
O_1	0	0	4	5	20
O_2	1	4	2	15	25
O_3	3	2	1	4	10
O_4	4	5	6	3	10
Demand	5	10	30	20	

As the allocation procedure is based on Least Cost Matrix (LCM) method, so for the comparison between LCM and WCDT based LCM approach, at first we will solve the problem with LCM method and then by the proposed virtual weighted cost based procedure.

Table 3. Final solution by the LCM method of the given TP problem

	D_1	D_2	D_3	D_4	Supply
O_1	0 **5**	0 **10**	4 ×	5 **5**	~~20~~,~~10~~,~~5~~
O_2	1 ×	4 ×	2 **20**	15 **5**	~~25~~,~~5~~
O_3	3 ×	2 ×	1 **10**	4 ×	~~10~~
O_4	4 ×	5 ×	6 ×	3 **10**	~~10~~
Demand	~~5~~	~~10~~	~~30~~, ~~20~~	~~20~~, ~~10~~,~~5~~	

It is observed that there are two minimal cost cells namely c_{11} and c_{12}, so choose arbitrarily one of the two cells for allocation. Let choose cell c_{12} and allocate

$min\{20,10\}$ = 10 to the cell. After continuing the process according to the rule of LCM method we have obtained the final solution which is displayed in the Table 3. Therefore the total cost of LCM method:

$$\mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$
$$=0\times5+0\times10+5\times5+2\times20+15\times5+1\times10+3\times10=\mathbf{180}$$

Now we will solve the problem by the virtual weighted cost based LCM procedure. According to the procedure we need first to formulate the virtual Weighted Cost Distribution Table (WCDT).

Solution: Step 1(Formulation of WCDT): Since there are two cells namely C_{11} and C_{12} having zero transportation cost. we need to find M first as follows:

$M = \max\{S_i\ , D_j\ \forall i,j\}$ ={20,25,10,10,5,10,30,20}=30

And here $\{c_{ij}: 0 < c_{ij} < 1\ \forall i,j\ \} \neq \Phi$. Therefore the virtual weighted cost corresponding to the zero cells cost :

$${}^{w}c_{11} = M\times\ min\ (S_1, D_1) = 30\times\ min\ (5, 20)$$
$$= 30 \times 5 = 150$$
$${}^{w}c_{12} = M\times\ min\ (S_1, D_1) = 30\times\ min\ (10, 20)$$
$$= 30 \times 10 = 300$$

and other weighted cell costs are given by the formula:

$${}^{w}C_{ij} = min\ (S_i, D_j) \times \frac{1}{c_{ij}}\ .$$

Therefore the complete virtual WCDT is given in the Table 4.

Table 4. Virtual Weighted Cost Distribution Table

	D_1	D_2	D_3	D_4	Supply
O_1	150	300	20/4	20/5	20
O_2	5/1	10/4	25/2	20/15	25
O_3	5/3	10/2	10/1	10/4	10
O_4	5/4	10/5	10/6	10/3	10
Demand	5	10	30	20	

For hand calculation, we will now incorporate this virtual WCDT into the given transportation cost table. After insertion, we have the virtual weighted cost based Transportation which is shown in the table 5. It is noted that each virtual weighted cost is given to the upper left corner of each cell whereas each actual cost is given to the upper right corner of each corresponding cell.

Table 5. Virtual weighted cost based Transportation table

	D_1	D_2	D_3	D_4	S
O_1	150 0	**300 0**	20/4 4	20/5 5	20
O_2	5/1 1	10/4 4	25/2 2	20/15 15	25
O_3	5/3 3	10/2 2	10/1 1	10/4 4	10
O_4	5/4 4	10/5 5	10/6 6	10/3 3	10
D	5	10	30	20	

Now we have to allocate to the cell according to the rule of LCM method but flow of allocation will be done according to the WCDT. That is, we will allocate to the cell which contains **largest virtual weighted cost**. It is observed in the table 5 that the cell C_{12} has largest virtual weighted cost namely 300, so we have to allocate to the cell C_{12} which obviously min {20, 10} i.e 10. So after first allocation, the table 7 shows the 1st allocated virtual weighted cost based Transportation Table. After first allocation, it is observed in the reduced table 7 (ignore column 2 as it satisfies all the demand) that the largest virtual weighted cost is 150 corresponds to the cell C_{11}, so we need to allocate in this cell now which is obviously min {10, 5} i.e. 5. Therefore, after second allocation we have Table 8. Now it is observed in the reduced table 8 (ignore column 1 and 2 as they satisfy all corresponding demand) that the remain largest virtual weighted cost is now 25/2 corresponds to the cell C_{23}, so we need to allocate in this cell now and which is obviously min {25,30} i.e.25. So after third allocation we have Table 8.

Table 7. After first allocation

	D_1	D_2	D_3	D_4	S
O_1	**150** 0	300 0 **10**	20/4 4	20/5 5	~~20~~,10
O_2	5/1 1	10/4 4 ×	25/2 2	20/15 15	25
O_3	5/3 3	10/2 2 ×	10/1 1	10/4 4	10
O_4	5/4 4	10/5 5 ×	10/6 6	10/3 3	10
D	5	~~10~~	30	20	

Table 8. After second allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4	20/5 5	~~20~~,~~10~~, 5
O_2	5/1 1 ×	10/4 4 ×	**25/2** 2	20/15 15	25
O_3	5/3 3 ×	10/2 2 ×	10/1 1	10/4 4	10
O_4	5/4 4 ×	10/5 5 ×	10/6 6	10/3 3	10
D	~~5~~	~~10~~	30	20	

Table 9. After third allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4	20/5 5	~~20~~,~~10~~, 5
O_2	5/1 1 ×	10/4 4 ×	25/2 2 **25**	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	**10/1** 1	10/4 4	10
O_4	5/4 4 ×	10/5 5 ×	10/6 6	10/3 3	10
D	~~5~~	~~10~~	~~30~~,5	20	

Similarly we have to allocate step by step according to the WCDT. The step by step allocation procedures are displayed on the tables 10 – 11 respectively and after completion of all allocation we have Table 12.

Table 9. After 4th allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4 ×	**20/5** 5	~~20~~,~~10~~, 5
O_2	5/1 1 ×	10/4 4 ×	25/2 2 **25**	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 **5**	10/4 4	~~10~~,5
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	10/3 3	10
D	~~5~~	~~10~~	~~30,5~~	20	

Table 10. After 5th allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4 ×	20/5 5 **5**	~~20~~,~~10~~, ~~5~~
O_2	5/1 1 ×	10/4 4 ×	25/2 2 **25**	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 **5**	10/4 4	~~10~~,5
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	**10/3** 3	10
D	~~5~~	~~10~~	~~30,5~~	~~20~~,15	

Table 11. After 6th allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4 ×	20/5 5 **5**	~~20~~,~~10~~, ~~5~~
O_2	5/1 1 ×	10/4 4 ×	25/2 2 **25**	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 **5**	**10/4** 4	~~10~~,5
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	10/3 3 **10**	~~10~~
D	~~5~~	~~10~~	~~30~~,5	~~20,15~~,5	

Table 12. After all allocation

	D_1	D_2	D_3	D_4	S
O_1	150 0 **5**	300 0 **10**	20/4 4 ×	20/5 5 **5**	~~20~~,~~10~~, ~~5~~
O_2	5/1 1 ×	10/4 4 ×	25/2 2 **25**	20/15 15 ×	~~25~~
O_3	5/3 3 ×	10/2 2 ×	10/1 1 **5**	10/4 4 **5**	~~10,5~~
O_4	5/4 4 ×	10/5 5 ×	10/6 6 ×	10/3 3 **10**	~~10~~
D	~~5~~	~~10~~	~~30~~,5	~~20,15,5~~	

Therefore the total cost of virtual WCDT based LCM method:

$$\mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$
$$=0\times5+0\times10+5\times5+2\times25+1\times5+ 4\times5+3\times10=\mathbf{130}$$

Now we have compared both the results obtained by the LCM method and the WCDT based LCM method respectively. The comparison is shown in the table 13. It is observed in the table 13 that the proposed WCDT based LCM approach outperforms the LCM approach.

Table 13. The comparison between LCM and WCDT based LCM approach

Method	**Total Cost**
LCM	180
WCDT based LCM	**130**

It is remarked that after second step of allocation, the classical LCM approach considered cell C_{33} for next allocation whereas for the present of virtual WCDT, the WCDT based approach considered cell C_{23} for third allocation. Eventually the classical LCM bounds to consider the cell C_{24} for last allocation but this cell contains higher transportation cost whereas WCDT based approach able to escape from this cell for allocation.

4. CONCLUSION

In the literature it is observed that the formulations of allocation flow are done by the manipulation of only cost entries. In this article, we have proposed a virtual Weighted Cost Distribution Table (WCDT) which is formulated by the demand or/and supply entries. The demand/supply entry of each cell is treated as weighted factor with some manipulation of the cell cost entry. We have implemented this WCDT in Least Cost Matrix method to control the flow of allocations. The procedure of allocations is demonstrated by a typical example. The elementary experimental results are nice, satisfactory and significance compared to LCM method.

Moreover, it is also hoped that, in future, researchers will able to obtain some nice approaches to solve TP by exploiting the concept of the proposed virtual WCDT.

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