

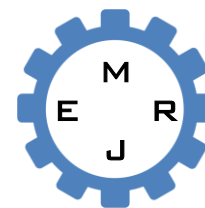


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PERFORMANCE OF NANOFLUID IN FREE CONVECTIVE HEAT TRANSFER INSIDE A CAVITY WITH NON-ISOTHERMAL BOUNDARY CONDITIONS

S. Parvin^{1,*}, R. Chowdhury^{1,2}, M. A. H. Khan¹ and M. A. Alim¹

¹Department of Mathematics, Bangladesh University of Engineering & Technology, Dhaka-1000, Bangladesh

²Department of Natural Science, Stamford University Bangladesh, Dhaka-1209, Bangladesh.

Abstract: The free convective flow and heat transfer characteristics of nanofluid inside a prismatic enclosure with non-uniform temperature distribution maintained at the bottom wall has been analyzed numerically. The water based nanofluid with alumina nanoparticle is used as the operational fluid through the enclosure. Finite Element Method using Galerkin's weighted residual scheme solves the governing partial differential equations with proper boundary conditions. Calculations are performed for different solid volume fraction $0 \leq \chi \leq 0.1$ with Rayleigh number 10^5 . An enhancement in heat transfer rate has observed with the increase of volume fraction of nanoparticles.

Keywords: Natural Convection, finite element method, Nanofluid, Solid Volume Fraction, Prismatic enclosure.

NOMENCLATURE

G	=	acceleration due to gravity [ms^{-2}]
K	=	thermal conductivity [$\text{Wm}^{-1}\text{K}^{-1}$]
L	=	length of the base and height of the prismatic cavity [m]
Nu	=	Nusselt number
p	=	dimensional pressure [Nm^{-2}]
P	=	dimensionless pressure
Pr	=	Prandtl number
Ra	=	Rayleigh number
T	=	temperature [K]
u, v	=	velocity components [ms^{-1}]
U, V	=	Dimensionless velocity components
x, y	=	distance along x - and y -coordinates [m]
X, Y	=	dimensionless distance along x - and y -coordinates
ν	=	kinematic viscosity of the fluid [m^2s^{-1}]
θ	=	dimensionless temperature

1. INTRODUCTION

Many engineering applications of natural convection in enclosures including geophysics, geothermal reservoirs, insulation of building, heat exchanger design, building structure etc. motivate many researchers to perform numerical simulation to investigate the flow pattern, temperature distribution, and heat flow. The low thermal conductivity of conventional heat transfer fluids, commonly water, has restricted designers. Fluids containing nanosized solid particles offer a possible solution to conquest this problem. The nanofluid has greater effective thermal conductivity than pure base fluid. Most of the available literature on this topic concerns regular geometries such as rectangular or square enclosures, while actual applications demand the consideration of irregular shapes. Literature reviews on natural convection inside triangular, trapezoidal and rhombic enclosures are available in [1-4]. In view of various applications of thermal processes, a comprehensive understanding of heat transfer and flow circulations within prismatic cavities is very much essential for industrial development. Current work attempts to analyze heat transfer and energy distributions using heatline concept inside a nanofluid filled prismatic cavity.

* Corresponding author: Email: salpar@math.buet.ac.bd

The heat recovery system or true path of convective heat transfer can be visualized by 'heat line' method. Kimura and Bejan [5] and Bejan [6] first introduced the concept of heat line. Heat lines represent heat flux lines which represent the trajectory of heat flow in the system and they are normal to the isotherms for conductive heat transfer. Various applications using heat lines were studied by Bello-Ochende [7], Costa [8-10], Mukhopadhyay et al. [11, 12] and Deng and Tang [13]. Dalal and Das [14] have used heat line method for the visualization of flow in a complicated cavity. Recently, Yaseen [15] has studied and analyzed numerically the steady natural convection flow in a prismatic enclosure with strip heater on bottom wall. For this enclosure, top inclined walls are considered at low temperature, two vertical walls were adiabatic and strip heater was at constant high temperature mounted on the bottom enclosure while the remainder bottom wall was kept at low temperature. Recently Ahmed et.al [16] performed numerical analysis for natural convection flows within prismatic enclosures based on heatline approach.

Heat transfer in cavities filled with nanofluid is the research interest of many researchers. Parvin et al. [17] studied the natural convection heat transfer in an enclosure with a heated body filled with nanofluid. Nasrin and Parvin [18] investigated numerically the buoyancy-driven flow and heat transfer in a trapezoidal cavity filled with water-Cu nanofluid. In their work, a correlation is developed graphically for the average Nusselt number as a function of the Prandtl number as well as the cavity aspect ratio. Abu-Nada and Chamkha [19] performed a numerical study of natural convection heat transfer in a differentially heated enclosure filled with CuO-EG-Water nanofluid. The results were compared with Brinkman model and MG models for nanofluid viscosity and thermal conductivity. Either enhancement or decline was reported for the average Nusselt number as the volume fraction of nanoparticles increased. Heat Transfer and Entropy Generation in an Odd-shaped Cavity filled with Nanofluid has analyzed by Parvin and Chamkha [20]. It must be noticed that, adding nanoparticles into the base fluid does not always increase its thermal conductivity. Teja et al. [21] have reported that the thermal conductivity of 2 nm titania nanoparticles is smaller than the thermal conductivity of the base fluid at the same temperature. Parvin et al. [22] numerically investigated the natural convection heat transfer from a heated cylinder contained in a square enclosure filled with water-Cu nanofluid. Their results indicated that heat transfer augmentation is possible using highly viscous nanofluid.

From the above discussion, the literature contains many investigations into the heat transfer performance of natural convection of nanofluid in regular shaped cavities. However, a comprehensive analysis on heat flow during natural convection of nanofluid in irregular enclosure with the heat line approach is yet to appear in the literature. Accordingly, the present study performs a numerical investigation into the natural convection heat flow within a prism shaped cavity containing nanofluid with non-isothermal bottom wall. The simulations focus specifically on the effects of the different volume fraction of nanofluid on the streamlines, isotherm distribution, heat lines and average Nusselt number. Average temperature and velocity are also presented. From this study, researchers will get experience to observe that how heat transfer rate is increasing or

decreasing in this model. As an irregular cavity has used in this simulation, it will allow to the researchers comparing the results between regular and irregular shaped models.

2. MATHEMATICAL FORMULATION

2.1 Momentum and Energy Formulation

The physical domain is shown in Fig. 1. The fluid properties are assumed to be constant except the density variation which is determined according to the Boussinesq approximation. Under these assumptions, the governing equations for steady two dimensional laminar incompressible flows can be written in dimensionless form as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + Pr \frac{v_{nf}}{v_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + Pr \frac{v_{nf}}{v_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \frac{(1-\chi)\rho_f\beta_f + \chi\rho_s\beta_s}{\rho_{nf}\beta_f} \theta \quad (3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

where, $\rho_{nf} = (1-\chi)\rho_f + \chi\rho_s$ is the density,

$(\rho C_p)_{nf} = (1-\chi)(\rho C_p)_f + \chi(\rho C_p)_s$ is the heat

capacitance, $\beta_{nf} = (1-\chi)\beta_f + \chi\beta_s$ is the thermal expansion

coefficient, $\alpha_{nf} = k_{nf} / (\rho C_p)_{nf}$ is the thermal diffusivity,

$\mu_{nf} = \mu_f (1-\chi)^{-2.5}$ is dynamic viscosity and

$k_{nf} = k_f \frac{k_s + 2k_f - 2\chi(k_f - k_s)}{k_s + 2k_f + \chi(k_f - k_s)}$ is the thermal conductivity

of the nanofluid and

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad \theta = \frac{T - T_c}{T_h - T_c},$$

$$P = \frac{pL^2}{\rho\alpha^2}, \quad Pr = \frac{\nu}{\alpha}, \quad Ra = \frac{g\beta(T_h - T_c)L^3 Pr}{\nu^2} \quad (5)$$

The boundary conditions can be summarized by the following equations:

- At the bottom wall: $U = 0, V = 0, \theta = \sin(\pi L)$
- For the side walls: $U = 0, V = 0, \frac{\partial \theta}{\partial X} = 0$
- For the inclined walls: $U = 0, V = 0, \theta = 0$

2.2 Stream function and Heat function

The relationships between stream function Ψ and velocity components U, V for two-dimensional flows are

$$U = \frac{\partial \psi}{\partial Y} \text{ and } V = -\frac{\partial \psi}{\partial X} \tag{6}$$

which give a single equation

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \tag{7}$$

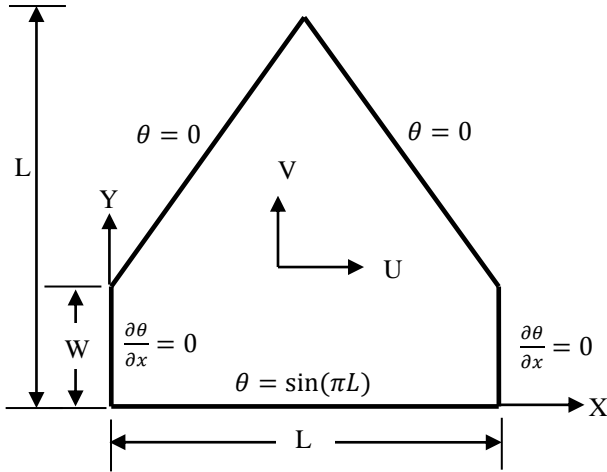


Fig. 1 Schematic diagram of the physical system.

The no-slip condition is valid at all boundaries as there is no cross-flow. Hence $\psi = 0$ is used for boundaries.

The heat flow within the enclosure is displayed using the heat function Π obtained from conductive heat fluxes $\left(-\frac{\partial \theta}{\partial X}, -\frac{\partial \theta}{\partial Y}\right)$ as well as convective heat fluxes $(U\theta, V\theta)$. The heat function satisfies the steady energy balance equation (Eq. (4)) such that

$$\frac{\partial \Pi}{\partial Y} = U\theta - \frac{\partial \theta}{\partial X} \text{ and } -\frac{\partial \Pi}{\partial X} = V\theta - \frac{\partial \theta}{\partial Y} \tag{8}$$

which yield a single equation

$$\frac{\partial^2 \Pi}{\partial X^2} + \frac{\partial^2 \Pi}{\partial Y^2} = \frac{\partial}{\partial Y}(U\theta) - \frac{\partial}{\partial X}(V\theta) \tag{9}$$

2.3 Average Nusselt Number

The average Nusselt number, average temperature and average velocity may be expressed as

$$Nu = -\frac{1}{S} \int_0^S \left(\frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial N} dN$$

$$\theta_{av} = \int \theta d\bar{V} / \bar{V} \text{ and } V_{av} = \int V d\bar{V} / \bar{V}$$

where S is the non-dimensional length of the surface and \bar{V} is the volume to be accounted.

3. NUMERICAL IMPLEMENTATION

The Galerkin finite element method is used to solve the non-dimensional governing equations along with boundary conditions for the considered problem. The equation of continuity has been used as a constraint due to mass conservation and this restriction may be used to find the pressure distribution. The finite element method of Reddy [23] is used to solve the Eqns. (6) - (9) where the pressure P is eliminated by a

constraint. The continuity equation is automatically fulfilled for large values of this penalty constraint. Then the velocity components (U, V) and temperature (θ) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ϵ such that $|\psi^{n+1} - \psi^n| \leq 10^{-4}$, where n is the number of iteration and ψ is a function of U, V and θ .

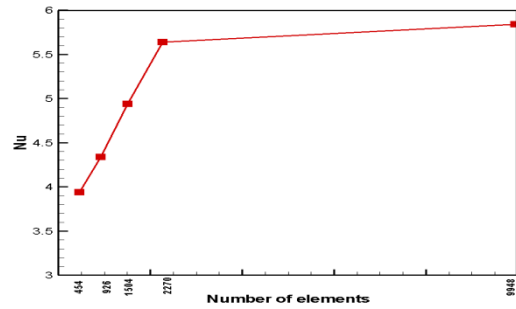


Fig. 2 Grid independent test.

3.1 Grid Independent Test

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for $Ra = 10^5$ and $Pr = 6.2$, $\chi = 0$ in a prismatic enclosure. Five different non-uniform grid systems with the following number of elements within the resolution field: 454, 926, 1504, 2270 and 9948 are examined. The numerical scheme is carried out for highly precise key in the average Nusselt (Nu) number to understand the grid fineness as shown in Fig. 2. The scale of the average Nusselt numbers for 2270 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 2270 elements is preferred for the computation.

4. RESULTS AND DISCUSSION

The numerical computation has been carried out through the finite element method to analyze natural convection within a nanofluid filled prismatic enclosure based on heatline concept. Results of isotherms, streamlines and heatlines for various values of nanoparticle volume fraction χ ($=0, 0.05, 0.1$) with Rayleigh number $Ra = 10^5$ and Prandtl number $Pr = 6.2$ in the prismatic enclosure are displayed. In addition, the values of average Nusselt number at the top inclined walls and the bottom wall as well as average temperature and average velocity in the cavity have been calculated for the mentioned parameter.

4.1 Effect on Isotherms

The influence of particle volume fraction χ on isotherms for the present configuration has been demonstrated in Fig. 3. The patterns of isotherms are smooth and are symmetric to the central vertical line for all the considered values of χ . A thermal plume rises from the middle of the bottom wall because of

sinusoidal boundary temperature. At $\chi = 0$, that is in the case of base fluid, the higher temperature lines remain near the hot bottom wall. These lines move toward the cold top inclined walls due to the increasing values of χ . This happens due to the presence of nanoparticle in the fluid and buoyancy effect which causes higher temperature gradient and thus the isothermal lines move from hot walls to the cold wall.

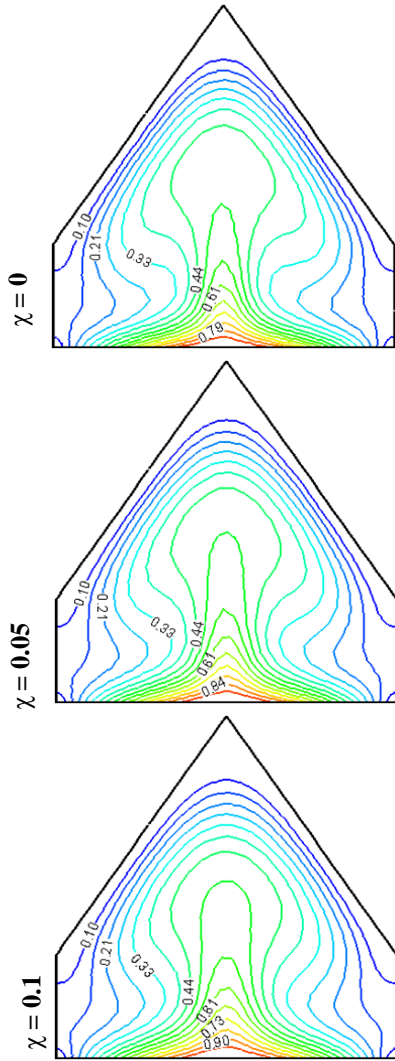


Fig. 3 Effect of χ on Isotherms.

4.2 Effect on Streamlines

Streamlines corresponding to different nanofluid solid volume fraction χ are shown in Fig. 4. It is seen from the figure that the trend of streamlines are similar for all cases. There are two symmetric circulation cells formed inside the enclosure. With the increasing value of the χ the streamlines are less dense near the central vertical line. It is noteworthy to mention that the central core of the circulatory cell is reduced in size for higher nanoparticle concentration. Higher flow intensity is seen for the base fluid.

4.3 Effect on Heatlines

The heatlines are constructed based on the thermal boundary conditions. As seen from the Fig. 5, the heatlines are

similar to streamlines at the core signifying convective heat flow and a large amount of heat flow occurs from the middle portion of the bottom wall as seen from dense heat lines. The two heat circulations in the system are observed and a very intense heat flow occurs across the middle of the cavity represented by dense heat line for pure water. It is interesting to observe that heat transport in a large regime at the core is due to convection. The large regime of convection is due to the large amount of heat transport from the bottom wall associated with large intensity of circulations. As the value of χ increases the convective heat transport has been suppressed.

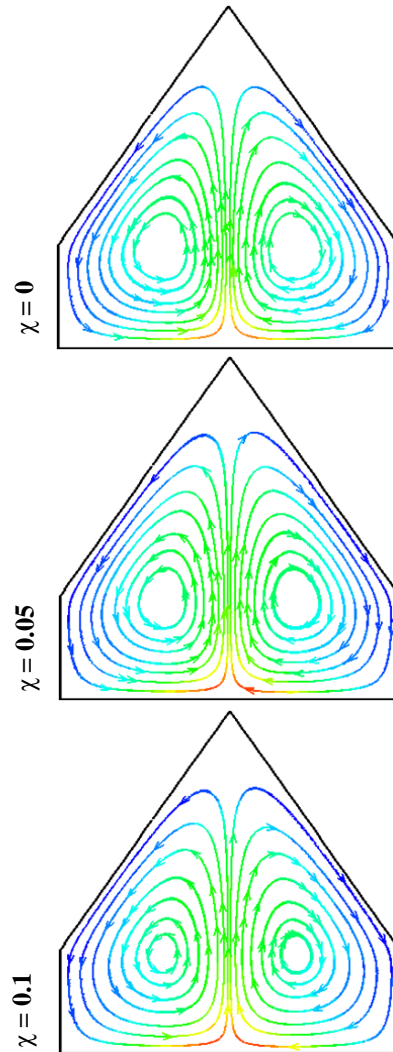


Fig. 4 Effect of χ on streamlines.

4.4 Average Nusselt Number

Fig. 6 shows the distribution of the average Nusselt number of the top inclined walls and bottom wall versus the solid volume fraction of nanofluid. General observation is that the average Nusselt number increases with nanoparticle concentration and it is utilized to represent the overall heat transfer rate within the domain. Similar trend of heat transfer is seen for both the walls. That is adding nanoparticles increases the thermal conductivity which leads to enhanced heat transfer.

4.5 Average temperature and average velocity

Fig. 7 demonstrates the distribution of average temperature

with the nanofluid volume fraction. It is seen that the average temperature decreases for increasing values of χ . The average velocity in the domain is depicted in Fig.8 which is also decreased by the increment of χ . The movement of the fluid becomes slower due to higher values of χ .

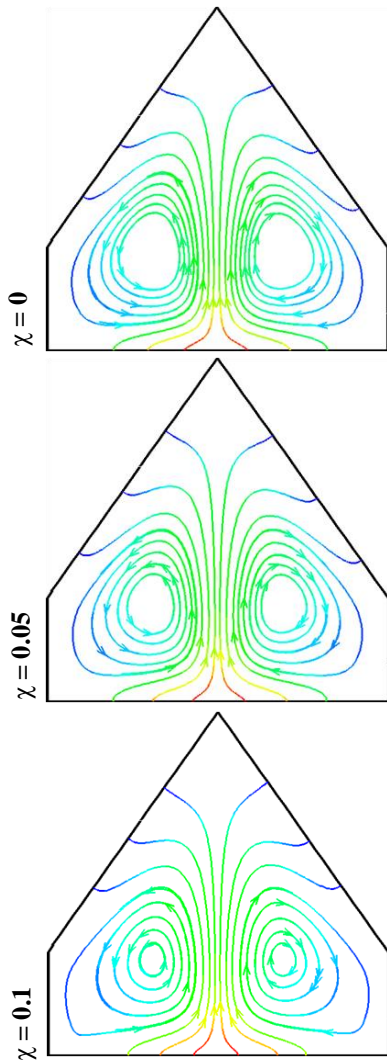


Fig. 5 Effect of χ on heat lines.

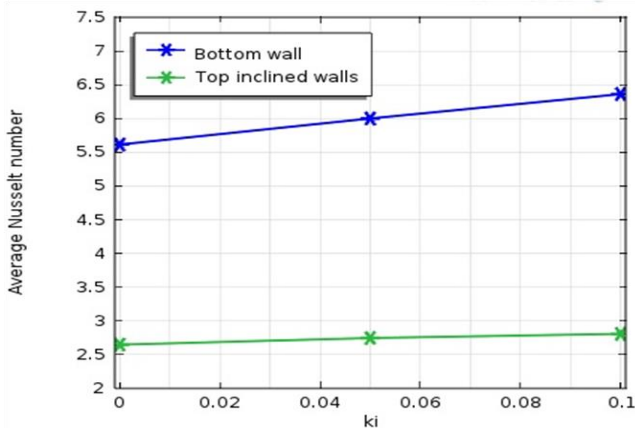


Fig. 6 Effect of χ on heat transfer.

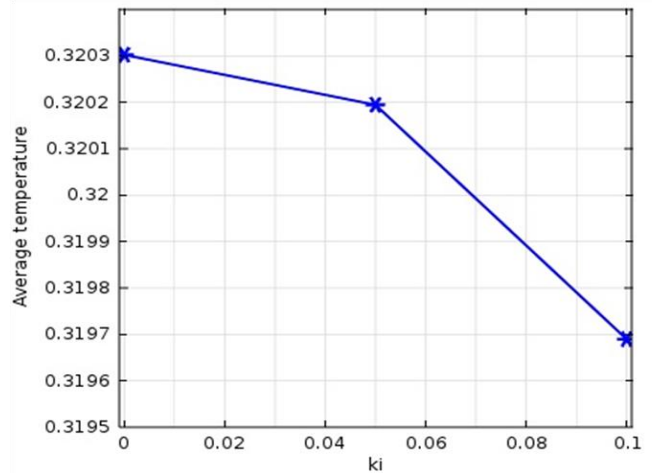


Fig. 7 Effect of χ on average temperature.

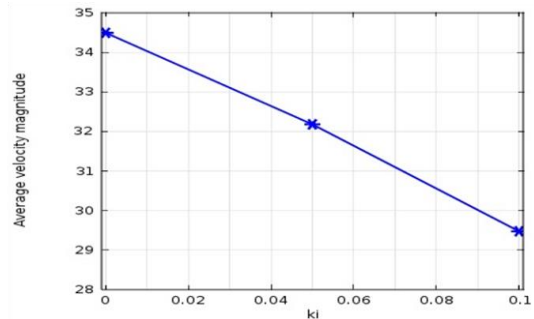


Fig. 8 Effect of χ on average velocity.

5. CONCLUSION

The prime objective of the current investigation is to analyze a physical as well as computational insight due to heat flow for natural convection within a prismatic enclosure filled with nanofluid. The key controlling parameter for this analysis is χ which govern the overall heat transfer rate, i.e. Nusselt number. The heat transfer rates are analyzed with average Nusselt number for the top inclined walls and the bottom wall of the enclosure. The motivation is to understand the effect of the parameter on the heat flow process. The visualization of heat flow inside any cavity is incomplete unless we know about the heat flow and hence we have introduced the heat line concept in the prismatic enclosure which enables us to understand the heat flow trajectory. In future, the present study can be expended by considering different boundary conditions and adding more effects on fluid flow, heat transfer and heat lines. The outcomes of the existing analysis are as follows:

- the solid volume fraction plays an important role on fluid flow pattern, heat transfer and heat lines.
- during conduction dominant heat transfer, it is observed that isotherms, streamlines and heat lines are found to be smooth. Also, the heat lines are seen normal to the isotherm during the heat transfer.
- as χ increases, convection is weakening. But heat transfer is increased due to conduction.

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