

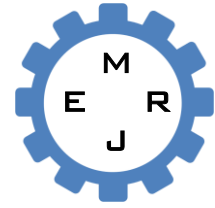


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## SIMULATION AND ANALYSIS OF HALF-CAR PASSIVE SUSPENSION SYSTEM

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**Abstract:** An independent front and rear vertical passive suspension is implemented on a half car model to simulate and analysis the reaction force exerted by the front and rear wheel due to pitch and bounce degrees of freedom of the car. MATLAB<sup>®</sup> Simulink<sup>®</sup> environment is used for numerical simulation of this model. This simulation provides a description about the ride characteristics of the model. A conventional passive suspension is used between the car body and wheel assembly which is made of a spring and a damper. The spring-damper characteristics are pre-selected to emphasize one of several conflicting objectives such as passenger comfort, road handling, and suspension deflection. In this model, the vehicle body pitch is represented by pitch angular displacement and pitch angular velocity and bounce degrees of freedom is represented by vertical displacement and vertical velocity.

**Keywords:** *Passive Suspension; Simulation; Half-Car; Pitch Moment; Bounce DoF.*

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### NOMENCLATURE

$F_f, F_r$  = upward force on the body from the front and rear suspension  
 $L_f, L_r$  = horizontal distance of center of gravity from front and rear suspension  
 $K_f$  = front suspension spring constant  
 $C_f, C_r$  = front and rear suspension damping rate  
 $Z, \dot{Z}$  = bounce distance (vertical) and its rate of change  
 $\theta, \dot{\theta}$  = pitch angle and its rate of change  
 $K_r$  = front suspension spring constant  
 $L = L_f + L_r$  = Wheelbase  
 $m_b$  = mass of the car  
 $I_{yy}$  = body moment of inertia about center of gravity  
 $M$  = pitch moment induced by car acceleration  
 $L_h$  = Length of the road hump  
 $H$  = Height of the road hump

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### 1. INTRODUCTION

Suspension is the system of springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two. Design and development of automotive suspension systems has been of great interest for nearly 100 years. Complicated vibration problems have arisen as a result of the increase in vehicle speeds which directly affect both the ride comfort and the ride safety. The solution of these problems in general may be achieved either by the reduction of the excitation level which mainly comes from the road surface irregularities or by the design of good suspension systems capable of maintaining an acceptable level of comfort and ensuring the vehicle safety on existing tracks [1]. The latter has been considered an important area of study and has been extensively investigated. The application of science to the problem has been increasing as time has passed.

Suspension is the system of springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two [2]. The primary purpose of the suspension system is to provide a high level of ride quality and protect the vehicle structure from harmful stresses by performing

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good isolation from the road surface irregularities. It also assure the lateral stability of the vehicle's road holding and controllability of braking at various running conditions (road qualities, speeds, accelerating and maneuvering) for good active safety, driving pleasure and keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps and vibrations etc.

## 2. SUSPENSION SYSTEMS

Most conventional suspensions use passive springs to absorb impacts and dampers (or shock absorbers) to control spring motions. Most important properties of a suspension are spring rate, damping factor, jacking forces, wheel rate, roll rate etc. Two of this properties are described below because a good understanding of these two is required for getting grasp on this work.

### 2.1 Spring Rate

The spring rate is a component in setting the vehicle's ride height or its location in the suspension stroke. The spring rate or spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring. Vehicles which carry heavy loads will often have heavier springs to compensate for the additional weight that would otherwise collapse a vehicle to the bottom of its travel (stroke). Springs that are too hard or too soft cause the suspension to become ineffective because they fail to properly isolate the vehicle from the road. However, the actual spring rates for a 2,000 lb. (910 kg) race car and a 10,000 lb. (4,500 kg) truck are very different.

### 2.2 Damping

Damping is the control of motion or oscillation, as seen with the use of hydraulic gates and valves in a vehicle's shock absorber. This may also vary, intentionally or unintentionally. Like spring rate, the optimal damping for comfort may be less than for control. Damping controls the travel speed and resistance of the vehicle's suspension. An undamped car will oscillate up and down. With proper damping levels, the car will settle back to a normal state in a minimal amount of time. Most damping in modern vehicles can be controlled by increasing or decreasing the resistance to fluid flow in the shock absorber.

### 2.3 Types of suspension

#### 2.3.1 Based on working principle

Suspensions are mainly classified into three groups by their working principles:

1. Passive suspensions: Traditional springs and dampers are referred to as passive suspensions. Most vehicles are suspended in this manner.
2. Semi-active suspensions: If the suspension is externally controlled then it is a semi-active or active suspension. Semi-active suspensions include devices such as air springs and switchable shock absorbers, various self-leveling solutions, as well as systems like hydro pneumatic, hydroelastic, and hydra gas suspensions.

Mitsubishi developed the world's first production semi-active electronically controlled suspension system in passenger cars [3].

3. Active suspension: Fully active suspension systems use electronic monitoring of vehicle conditions, coupled with the means to impact vehicle suspension and behavior in real time to directly control the motion of the car.

#### 2.3.2 Based on wheel's motion

Suspension systems can be broadly classified into three subgroups by the ability of opposite wheels to move independently of each other:

1. Dependent: A dependent suspension normally has a beam (a simple 'cart' axle) or (driven) live axle that holds wheels parallel to each other and perpendicular to the axle.
2. Independent: An independent suspension allows wheels to rise and fall on their own without affecting the opposite wheel. Suspensions with other devices, such as sway bars that link the wheels in some way are still classed as independent. In 1922, independent front suspension was pioneered on the Lancia Lambda and became more common in mass market cars from 1932 [4].
3. Semi-dependent: In this case, the motion of one wheel does affect the position of the other but they are not rigidly attached to each other. A twist-beam rear suspension is such a system.

## 3. METHODOLOGY

The modeled characteristics of the half-car are illustrated in Fig.1. The front and rear suspension are designed as spring and damper systems like a conventional suspension.

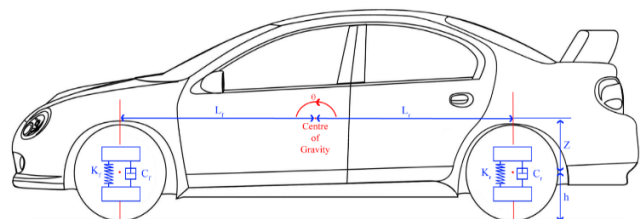


Fig. 1 Diagram of half car model with suspensions.

A spring is characterized by its stiffness ( $k$ ). According to the Hooke's law, a force ( $F_s$ ) to generate a deflection in spring is proportional to relative displacement of its ends [5]. The stiffness  $k$  may be a function of position and time. If  $k$  is constant then, the value of stored potential energy in the spring is equal to the work done by the spring force  $F_s$  during the spring deflection. The spring potential energy is then a function of displacement. If the stiffness of a spring,  $k$ , is not a function of displacements, it is called linear spring. Force exerted by the spring of the front suspension due to the bounce of the car is shown by the

following Eq. (1)

$$F_{fs1} = -K_{fs}Z \quad (1)$$

Suspension spring extends or compresses by a distance  $L_f\theta$  due to induced pitch moment of the car. Hence, force exerted because of this effect can be described by Eq. (2)

$$F_{fs2} = K_f L_f \theta \quad (2)$$

Total force exerted on the spring is

$$F_{fs} = F_{fs1} + F_{fs2} = -K_{fs}Z + K_f L_f \theta \quad (3)$$

Damping of a damper is measured by the value of mechanical energy loss in one cycle. Equivalently, a damper may be defined by the required force  $F_{fc}$  to generate a motion in the damper. If  $F_{fc}$  is proportional to the relative velocity of its ends, it is a linear damper with a constant damping  $C$ . Such a damping is also called viscous damping which is modeled by the following equation.

$$F_{fc} = -C_f \dot{Z} \quad (4)$$

Just like Eq. (3), we can derive an equation for total force exerted on the damper of the front suspension of the car.

$$F_{fc} = -C_f \dot{Z} + C_f L_f \dot{\theta} \quad (5)$$

Total force exerted on the front suspension is obtained by Eqs. (3) & (5)

$$F_f = 2K_f(L_f\theta - Z) + 2C_f(L_f\dot{\theta} - \dot{Z}) \quad (6)$$

The front suspension influences the bounce (i.e. vertical degree of freedom) according to Eq. (6).

Force exerted by the rear suspension on the car is expressed by Eq. (7), which also takes a part to affect the bounce.

$$F_r = -2K_r(L_r\theta - Z) - 2C_r(L_r\dot{\theta} - \dot{Z}) \quad (7)$$

The pitch contribution to the front suspension is given by Eq. (3)

$$M_f = -F_f L_f \quad (8)$$

The pitch contribution to the rear suspension is given by Eq. (4)

$$M_r = F_r L_r \quad (9)$$

Both of this moment is considered with respect to center of gravity. So, we can derive a relation between moment of inertia of the body and rate of change of pitch angle which is shown in the following Eq. (10).

$$I_{yy}\ddot{\theta} = M_f + M_r + M \quad (10)$$

Taking moment with respect to center of gravity of all forces, the following relation is obtained.

$$m_b \ddot{Z} = F_f + F_r - m_b g \quad (11)$$

#### 4. MODEL DESIGNING

The model designed in MATLAB Simulink are based on the relations described in methodology section. There are four dependent variables ( $F_f, F_r, M_f, M_r$ ) in Eqs. (6) to (9). Those equation can be used to find the forces & moments acting on the suspension and it is shown in Fig. 6. The suspension model shown in Fig. 6 has two inputs. The first input is the road height. A subsystem input here corresponds to the vehicle driving over a road/speed hump with changes in height. The dimensions of a bump will range from 30 to 90 centimeters wide & 3 to 10 centimeters high.

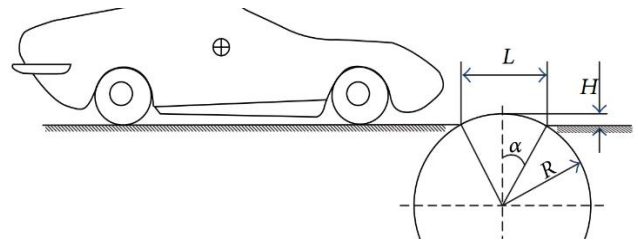


Fig. 2 Road hump geometry.

According to the Fig. 2 the following Eqs. (12) & (13) is obtained.

$$R = L_h / 2 \sin \alpha \quad (12)$$

$$R = H / 1 - \cos \alpha \quad (13)$$

According to The Highways (Road Humps) Regulations 1999 [6], putting length of the hump  $L_h = 0.9\text{m}$  and  $H = 0.1\text{m}$  in the above equations, we obtain  $R = 1.0627\text{m}$  &  $\alpha = 25.05^\circ$ . According to [7], optimum hump crossing speed of an automobile of that standard described above is 15km/h to 25km/h. Let, the speed ( $v$ ) of the car is 15km/h, the car will take approximately 0.3s to pass the hump. Several integrators and step change is used to generate a circular road profile by using the values stated above. The diagram is shown in Fig. 3.

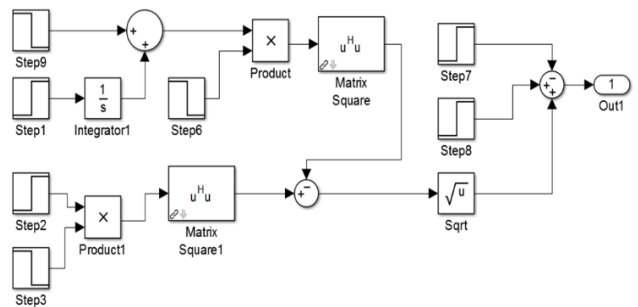


Fig. 3 Sub system to generate circular road profile.

As bounce distance ( $Z$ ) is influenced by the road height, the actual bounce of the car is  $Z_{actual} = Z + h$ . This bounce ( $Z_{actual}$ ) is used for calculation in the model. The road profile is shown Fig. 4.

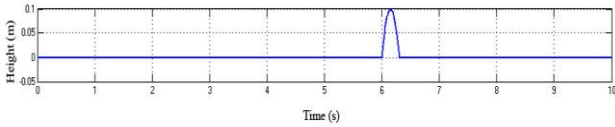


Fig. 4 Road profile. The car will experience a road hump at t = 6s.

The second input is a horizontal force acting through the center of the wheels those results from braking or acceleration maneuvers. This input appears only as a moment about the pitch axis because the longitudinal body motion is not modeled.

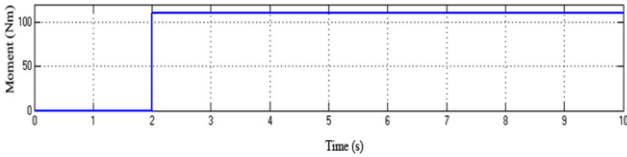


Fig. 5 Moment induced by the acceleration of a car. The car start running at t = 3s, as a result a pitch of 133 Nm is induced on the car.

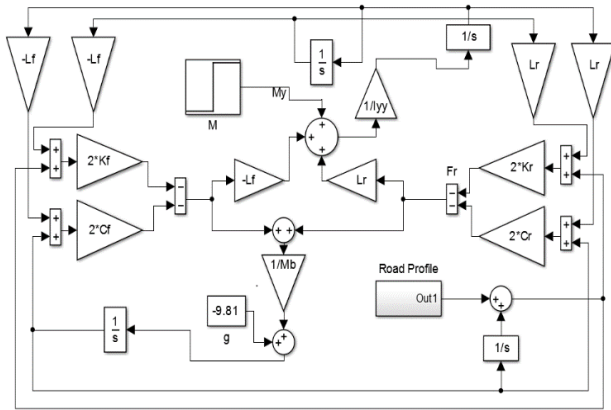


Fig. 6 Top-level diagram of the passive car suspension model.

The whole half car suspension system is modeled by using the Eqs. (6) to (11) in SimuLink environment. The equations are implemented directly in the Simulink® diagram through the straightforward use of Gain, Integrator and Summation blocks.

A different data set ( $L$ ,  $K$  and  $C$ ) can be entered for each instance. Furthermore,  $L$  is thought of as the Cartesian coordinate  $x$ , being negative or positive with respect to the origin, or center of gravity. Thus,  $K_f$ ,  $C_f$  &  $L_f$  are used for the front suspension block whereas  $K_r$ ,  $C_r$  &  $L_r$  are used for the rear suspension block.

### 5. SIMULATION RESULT

The specification of the vehicle are taken from the car Toyota Avalon XLE 2015 [8]. These specification are introduced in the Simulink model by uploading the variables in workspace of MATLAB from M file editor. The results are viewed and plotted by using scope block. The default initial conditions and specifications are given below:

Wheel base of the car,  $L = 111$  in or  $2.88$  m

Front hub displacement from body gravity center,  $L_f = 1.23$  m

Rear hub displacement from body gravity center,  $L_r = 1.65$  m

Body mass,  $m_b = 3461$  lb. or  $1570$  kg

Suspension stiffness and damping coefficient are as follows:

Front suspension stiffness,  $K_f = 36400$  N/m

Rear suspension stiffness,  $K_r = 27300$  N/m

Front suspension damping,  $C_f = 3250$  Ns/m

Rear suspension damping,  $C_r = 2600$  N sec/m

Body moment of inertia about y-axis,  $I_{yy} = 2730$  kg m<sup>2</sup>

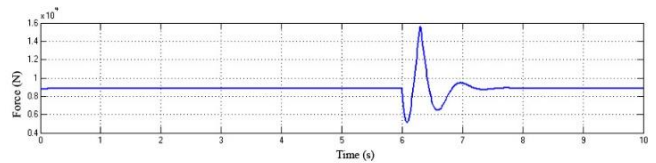


Fig. 7 Reaction forces at the front wheel.

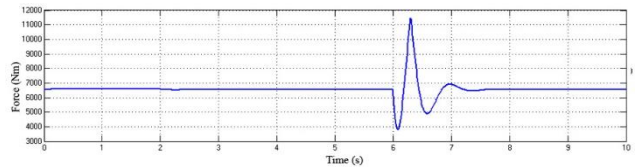


Fig. 8 Reaction forces at the rear wheel.

Form the Fig. 7 and Fig. 8, it can be inferred that there are little changes in the forces (almost negligible) applied on the wheels (both front and rear) when a moment is induced at the time of the car start moving (at 2s in the timeline of the 10s simulation along x-axis). But substantial amount of changes in the forces are experienced when the car is moving over the hump. It is shown at 6s to 8s after the simulation starts.

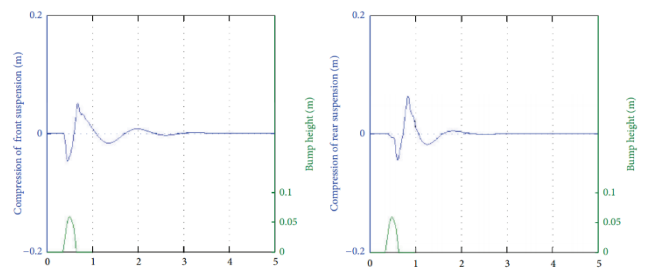


Fig. 9 Vertical dynamics results from a research article on Bump Modeling and Vehicle Vertical Dynamics Prediction [9].

Comparing Fig. 9 with Fig. 7 & Fig. 8 to validate the simulation result of this study. Fig. 9 reflects the change of length of the spring of the suspension. According to Hooke's law, it can be stated that force exerted on the spring is proportional to its change in length. Compression of the suspension in his simulation is proportional to the corresponding forces shown in Fig. 7 & Fig. 8. The curves in the both figure are similar in shape to each other. Since Fig. 7 and Fig. 8, represent forces and Fig. 9

represents distances, there are no similarity in magnitude in these two graph as expected. The curves in Fig. 9 are less smooth because of using dampers of high magnitude.

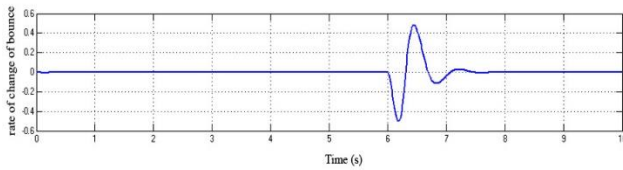


Fig. 10 Rate of change of bounce disturbance.

The car bounces up and down from its equilibrium point as the spring oscillate. The rate of change in bounce (i.e. the rate of change of the vertical distances the car travels) is depicted in the Fig. 10. At the beginning, there exist a little disturbance in Fig. 10 which is not expected in the real life scenario. It is visible more clearly in the following figure.

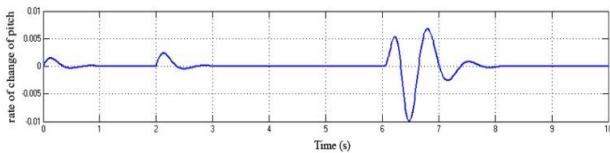


Fig. 11 Rate of change of pitch angle.

The possible reason of this change in pitch and bounce is the exchange of force and momentum between the front and rear suspensions. Another plausible reason of this behavior is the inaccurate implementation of initial values and car specifications. Solving this problem requires cumbersome trial and error method. Matter of hope is that, since the simulation reach to stability as soon as possible, this problem is no big issue in this simulation. If Fig. 11, when the car starts running, pitch moment has been induced after 2 seconds, because of the acceleration of the car. Even when the car experienced the road disturbance, it also pitch moment is induced in the system which is visible after 6s as the car started moving over the hump at 6 seconds in the simulation.

## 6. CONCLUSION

In this work, we mainly concentrate on simulating and understanding the behaviors of the conventional mechanical spring and damper system. By using this model, the effects of changing the suspension damping and stiffness can be simulated, thereby investigating the tradeoff between comfort and performance. It is known to all of us that racing cars have very stiff springs with a high damping factor, whereas passenger vehicles have softer springs and a more oscillatory response. We can evaluate this type of conclusions by observing the oscillations of bounces and pitch moments. This model also can be used in conjunction with a powertrain simulation to investigate longitudinal shuffle resulting from changes in throttle setting. A more detailed model would include a tire model, and damper nonlinearities such as velocity-dependent damping (with greater damping during rebound than compression). A full model

with six degrees of freedom also can be implemented using vector algebra blocks to perform axis transformations and force/displacement/velocity calculations. A further extension of this work can be the development of an active suspension by using a force actuator in the both suspension system. A non-linear or PID controller might be designed for the actuator as well.

## REFERENCES

- [1] A. S. Hasan, Fundamental studies of passive, active and semi - active automotive suspension systems, pp.2, University of Leeds Press, Leeds, July 1986.
- [2] R. N. Jazar, Vehicle Dynamics: Theory and Applications, pp.45, Riverdale, Springer Science+ Business Media, New York, 2008.
- [3] Technology DNA of MMC, Mitsubishi Motors Technical Review, 2005.
- [4] K. K. Jain and R. B. Asthana, Automobile Engineering, 1st edition, pp. 293–294, Tata McGraw-Hill, London, 2002.
- [5] A. C. Ugural and S. K. Fenster, Advanced Strength and Applied Elasticity, 4th ed. Prentice Hall PTR, New Jersey, 2003.
- [6] Statutory instruments, Highways, England and Wales, The Highways (Road Humps) Regulations 1999.
- [7] P. A. Weber and J. P. Braaksma, Towards A North American Geometric Design Standard For Speed Humps, Institute of Transportation Engineers (ITE), Washington D.C., USA, January 2000.
- [8] Brochures of Toyota Avalon, [Online] Available: <http://www.toyota.com/avalon/ebrochure>, Retrived on 15/04/15.
- [9] D. Garcia-Pozuelo, A. Gauchia, E. Olmeda and V. Diaz, "Bump modeling and vehicle vertical dynamics prediction." Advances in Mechanical Engineering, Vol. 6, 736576, 2014.