Dept. of Mech. Eng.
CUET

Published Online March 2019 (http://www.cuet.ac.bd/merj/index.html)

Mechanical Engineering Research Journal

Vol. 11, pp. 19–24, 2018

ISSN: 1990-5491

INVESTIGATION OF MODE I STRESS INTENSITY FACTOR FOR THE ECCENTRIC HOLE WITH CRACKS ON A THIN PLATE

T. H. Pial[1], O. Bhowmik[1], M. Shuvo[1], R. H. Kabir[1] and A. A. Amin[2*]

[1]Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Bangladesh
[2]Department of Mechanical and Aerospace Engineering, Case Western Reserve University, USA

Abstract- Thin plate with an eccentric hole containing crack is a more common machine element in contrast to central hole crack element. In this investigation, stress intensity factor (SIF) due to mode–I failure (KI) of such plate is calculated with finite element analysis (FEA). Displacement extrapolation and J-integral methods are employed to calculate the SIF values using quadratic 2D elements. The eccentricity of the hole on a thin plate is varied along with crack length. Changes in SIF are recorded with varying normalized hole eccentricity and normalized crack length. Result suggests the SIF values increase with increasing crack length and increasing eccentricity of the hole on the plate. The results are compared with the analytical solution for cracked thin plates. The investigation concludes analytical solutions for cracked plate is a good estimation of SIF values for cracked plate with an eccentric hole.

***Keywords**: Mode-I failure, Stress Intensity Factor, Displacement extrapolation, J-integral.*

NOMENCLATURE

a	Crack length
b	Plate half width
h	Plate half length
R	Hole radius
σ	Normal stress
E	Young modulus
G	Shear modulus
J	J-integral
K_1	Stress intensity factor for mode I crack
U	Nodal displacement parallel to crack growth direction
V	Nodal displacement perpendicular to crack growth direction
W	Strain energy density
v	Poisson's ratio
Γ	Arbitrary contour surrounding crack tip

1. INTRODUCTION

In structural components, stress concentration zones like flaws, notches, reentrant corners and holes are the main reasons behind crack initiation and growth. Static and fatigue failure occurs due to these cracks, so understanding the intensity of these cracks is necessary for sustainable structural components. In linear elastic fracture mechanics (LEFM), stress intensity factor (SIF) is an essential parameter which gives an idea about the severity of cracks and singular stress. Crack tip SIF is useful to predict the behavior of fatigue crack propagation which reflects the effect of applied load, crack and component geometry.

Among different types of crack loading, mode I is the most common crack loading in engineering applications. SIF for mode I loading is expressed as K_I which represent crack opening mode. By using Paris's law [1] we can calculate K_I for general conditions which are available in handbooks [2,3]. These theoretical solutions are limited to simple geometry and loading conditions. However, in real engineering, geometries are complicated, and loading conditions are also complex. Analytical calculations require extensive reworking for such complex scenarios. Numerical techniques such as Finite Element Analysis (FEA) provides an edge over analytical

[*]**Corresponding author:** Email: abdullah.amin@case.edu

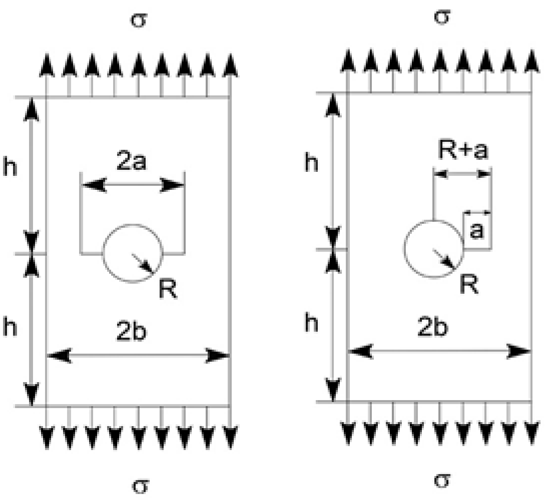
solution for calculating stress intensity factors for relatively complex geometry, loading, and boundary conditions. FEA has been applied successfully to study the failure of many different failure conditions previously. In some of these study, a regular standard case has been explored [4-6] and elaborated to estimate the SIF values. Complex geometric shapes such as three-dimensional cracks in piezoelectric structures [7], bi-material interface [8], and bridge roller bearings [9] were successfully analyzed using FEA. FEA offers different approaches to obtain stress intensity factor–displacement extrapolation method [10,11], stress extrapolation method, and j-integral [12,13] method are commonly used. These methods use displacement, stress or energy in the elements around the crack tip to evaluate SIF, and is highly dependent on element type and size.

In this work, structural steel plates with an eccentric hole containing cracks on one side or both sides are considered. Stress intensity factor (SIF) due to mode–I failure of such plate is calculated with FEA. Displacement extrapolation and J-integral methods are employed to calculate the SIF values with quadratic elements. The eccentricity of the hole on the thin plate is varied. Crack length is also varied as the hole diameter is kept constant throughout the investigation. A comparison with theoretical solutions is made for two cases to estimate the agreement between FEA and analytical calculations.

2. PROBLEM SPECIFICATION AND ANALYTICAL SOLUTION

In this study, we assume that cracks emanate from the hole periphery and grow along the horizontal direction. The eccentric hole is introduced on the plate and double cracks propagated in the opposite direction from the hole periphery for the plate with two cracks. We identify the crack near to the center line of the plate as inner crack and the crack near the edge of the plate as outer crack. When eccentricity is zero, the hole is formed in the center of the plate, and so inner and outer cracks have no significant difference. Due to mode-I failure, a thin plate with an eccentric circular hole containing crack at one outer side, one inner side and both outer and inner sides are studied. Here, the full width of the cracked plate is $2b = 200\ mm$, and length, $2h = 600\ mm$. The radius of the circular hole is $R = 6\ mm$. For the analytical solution, 6 mm long crack with zero eccentricity is considered as a similar case is available in the handbook [2,3]. The material of the plate is considered structural steel (ASTM-A36) with Young Modulus, E = 2×10^5 MPa, Poisson's ratio, ν = 0.30. The thin plate is subjected to uniform tensile stress along the Y- axis and the amplitude is σ=200MPa as shown in Fig. 1.

SIF at the crack tip for a plate under tensile loading with circular hole and crack is given by $K_I = A\sigma\sqrt{\pi a}$ [2,3]. Where, 'A' is a parameter called stress intensity modification factor associated with the crack geometry and the boundary conditions. For dual cracks 'a' is the summation of crack lengths and hole's radius on the other hand for single crack, 'a' is considered as the crack length, as shown in Fig. 1. Theoretical SIF solutions at the crack tip of the dual cracks and single crack shown in Fig. 1 are listed in Table 1.



(a) Dual cracks (b) Single crack

Fig. 1 Geometries of cracks emanating from hole.

Table 1 Analytical solutions of SIF for dual cracks and single crack.

	Dual cracks	Single crack
A	≈1	≈ 1.27
K_I ($MPa\sqrt{mm}$)	1228.1	1102.76

3. THEORETICAL BACKGROUND

3.1 Displacement Extrapolation Method

The displacement extrapolation method uses nodal displacements values around the crack tip to calculate stress intensity factor. This method can be formulated with crack tip displacement equations [12] as

$$U = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\cos\left(\frac{\theta}{2}\right)\left[1 - 2v + sin^2\left(\frac{\theta}{2}\right)\right]$$
$$V = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\sin\left(\frac{\theta}{2}\right)\left[1 - 2v + cos^2\left(\frac{\theta}{2}\right)\right] \qquad (1)$$

Here U and V are the displacements along parallel and perpendicular to the crack direction of node A respectively; K_I is the stress intensity factor; G is shear modulus. For plane strain conditions $G=\frac{E}{2(1-v)}$ where E is the Young modulus and v is the Poisson ratio. Assuming A is a node close enough to crack tip and expressed with r and θ as shown in Fig. 2. When calculating the perpendicular displacement, $\theta = \pi$ and SIF can be calculated as Eq. (2)

$$K_{I^*} = \frac{E}{4}\sqrt{\frac{2\pi}{r}}V \qquad (2)$$

From the plot of K_{I^*} as a function of r for $\theta = \pi$, estimation of K_I can be made by extrapolating the plot at r = 0, which indicates the crack tip. Extrapolation is necessary because numerical iteration experience singularity exactly at the crack tip node.

3.2 J-Integral Method

For calculating SIF values by the J-integral method, values of J-integral is calculated first. J-integral is the value of a contour integral characterizing the strain energy release rate for a nonlinear elastic material. J. R. Rice [10]. It is defined as an integral which is independent of integration contour Γ around a crack tip as in Eq. (3).

$$J = \int (W dy - T.\frac{\delta u}{\delta x} ds)_{\Gamma} \tag{3}$$

Where W is the strain energy density, T is the traction vector on a plane defined by the outward normal $(T = \sigma n)$, u is the displacement vector and y is the direction perpendicular to the crack line. Γ is an arbitrary contour surrounding the crack tip as shown in Fig. 2. In linear elastic fracture mechanics (LEFM), J-integral is proportional to the square of the crack tip stress intensity factor as the strain energy release rate is equal to the strain energy release rate along a contour at crack tip vicinity. For plane strain and plane stress conditions, the stress intensity factor can be calculated

$$J = {K_I}^2 / E(1 - v^2) \tag{4}$$

$$J = {K_I}^2 / E \tag{5}$$

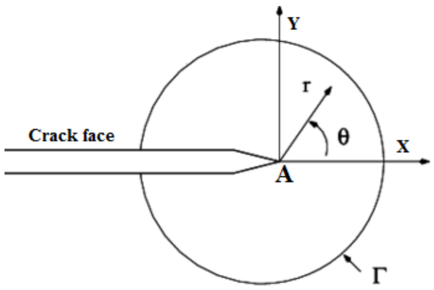


Fig. 2 Crack tip coordinates and contour.

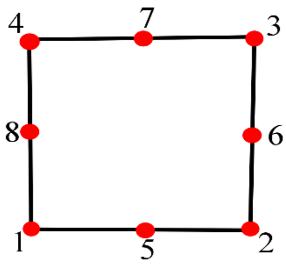


Fig. 3 Eight node quadratic element.

4. FINITE ELEMENT MODELING

Finite element analysis of our cracked plates was conducted with finite element software ANSYS [15]. We adopted quadratic element Plane-183 as our solid structural element (Fig. 3) [16]. Element size 1.00 mm was assigned for all eccentricity and crack length. The model was meshed with 181321 nodes and 60138 elements. Due to the symmetry of the geometry under investigation, half of the plate is considered for FEA analysis. Uniform tensile stress was applied perpendicular to the crack growth direction. We used mapped mesh for both displacement extrapolation and j-integral method, where stress concentration point was the crack tip point. A meshed pattern for two crack condition before loading is shown in Fig. 4(a). The special singular element is implemented on the crack (left crack in Fig. 4) under investigation. The other crack end is left with regular rectangular elements. After loading, cracked plate deformed and is shown in Fig. 4(b). Stress distribution around the crack tip is shown in Fig. 5. As expected, stress was concentrated around the crack tip region.

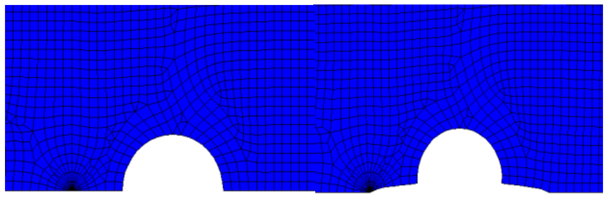


Fig. 4 A meshed pattern for dual crack condition, (a) before loading, (b) after loading.

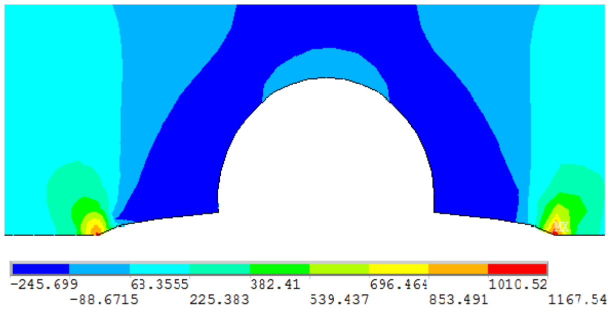


Fig. 5 Stress distribution (in MPa) around crack tips, colors follow the stress concentration values.

5. RESULTS AND DISCUSSION

For varying eccentricity, the position of the circular hole is varied as the diameter of the hole and the length of cracks were kept fixed. Fixed diameter was 12 mm and fixed crack length was 6 mm. Figure 6(a) shows results for normalized eccentricity obtained by displacement extrapolation method (eccentricity is normalized by half plate width). The figure shows that the stress intensity factor for dual cracks is larger compared to single crack. For the normalized eccentricity value of 0.00 (center hole), SIF for both crack tip points of dual cracks are around $1260 MPa\sqrt{mm}$. The value remain nearly constant for eccentricity value of 0.465. After that, change in SIF values are significant and start to increase with increasing eccentricity reaching at 1602 $MPa\sqrt{mm}$ for the outer crack tip and 1439 $MPa\sqrt{mm}$ for the inner crack tip at eccentricity 0.836. Single cracks follow the same pattern with lower values of SIF. For center hole, $K_I = 1115$ $MPa\sqrt{mm}$ for both crack tips and these remain nearly constant for eccentricity values of 0.465. Then changes in K_I is significant and follows a parabolic trend with final values of 1185 $MPa\sqrt{mm}$ and 1353 $MPa\sqrt{mm}$ for inner and outer crack tip respectively.

Figure 6(b) represents finite element analysis results for variable eccentricity obtained by the J-integral method. This figure shows a similar behavior as Fig. 6(a). As expected, K_I

values for dual cracked plate is much higher than single crack. For no eccentricity, K_I values are around 1290 $MPa\sqrt{mm}$ for both of the cracks. But after eccentricity cross 0.465, K_I start to increase with increasing eccentricity which reach at 1570.4 $MPa\sqrt{mm}$ for the outer crack tip and 1473.2 $MPa\sqrt{mm}$ for the inner crack tip at eccentricity values of 0.836. In absent of eccentricity, $K_I = 1142\ MPa\sqrt{mm}$ for both single crack tips and these remain nearly constant for eccentricity value of 0.465. Then K_I reach final values of 1214 $MPa\sqrt{mm}$ and 1312.9 $MPa\sqrt{mm}$ for inner and outer crack tip respectively.

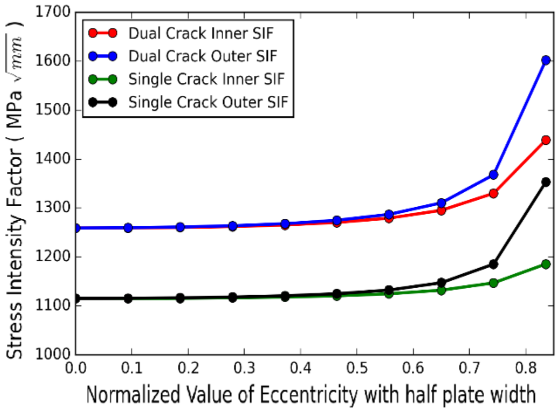


Fig. 6(a) SIF values for different eccentricity with fixed crack length of 6 mm. (Displacement Extrapolation method).

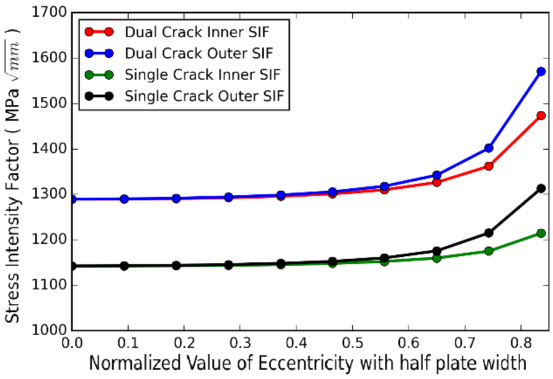


Fig. 6(b) SIF values for different eccentricity with fixed crack length of 6 mm (J-integral method).

For investigating the response of stress intensity factor on crack length, we took 5 different normalized values of crack length (by dividing crack length with hole diameter 12mm) for each of the four different types of crack tip. From displacement extrapolation method with 1.00 normalized value of crack length, our obtained K_I for the inner crack tip with dual crack is 1558.3 $MPa\sqrt{mm}$ and for the outer crack tip with dual crack is 1579.8 $MPa\sqrt{mm}$. Maintaining a linear increasing pattern, these values reaches at 2133.7 $MPa\sqrt{mm}$ and 2290.2 $MPa\sqrt{mm}$ respectively. As expected, for single crack systems K_I is much lower than dual cracked plates. For the inner crack tip of single crack system K_I =1259.9$MPa\sqrt{mm}$ and for outer crack tip K_I =1277.3 $MPa\sqrt{mm}$. SIF increases in a linear manner and for normalized value of crack length 2.0, $K_I = 1537.98\ MPa\sqrt{mm}$ and $K_I = 1643.18\ MPa\sqrt{mm}$ respectively. These results are summarized in Fig. 7(a).

Obtained results from J-integral method are shown in Fig. 7b. For normalized value 1.00, we get K_I for the inner crack tip with dual cracks is 1593.974$MPa\sqrt{mm}$ and for the outer crack tip with dual cracks is 1615.845 $MPa\sqrt{mm}$. Maintaining a linear increasing pattern these values reach to 2183.7 $MPa\sqrt{mm}$ and 2342.9 $MPa\sqrt{mm}$ respectively. For single crack system K_I is much lower than dual cracked plates. For inner crack tip of single crack K_I =1287.908$MPa\sqrt{mm}$ and for outer crack tip K_I =1305.657 $MPa\sqrt{mm}$. SIF increases in a linear manner and for normalized value of crack length 2.0, simulation says $K_I = 1572.76MPa\sqrt{mm}$ and $K_I = 1680.42MPa\sqrt{mm}$ respectively. The analytical solution without eccentricity is around 1228.1 $MPa\sqrt{mm}$ for dual cracked plate and 1102.76 $MPa\sqrt{mm}$ for single cracked plate. Results obtained from two different finite element methods are compared with analytical solution and summary of the comparison is shown in Table 2. Displacement extrapolation method agree more closely with analytical approximation than j-integral method. We get 2.6% and 1.11% errors for dual cracked plate and single cracked plate respectively by displacement extrapolation method which are 5.04% and 3.55% respectively for j-integral method. It is also noticeable that single cracked system is less erroneous than dual cracked system. In our study element size is 1mm, which is reasonably fine mesh for accurate result. Reducing element size may increase the efficiency but at a cost of more computing effort.

Trends of K_I can be explained by using finite size effect on stress concentration [17]. In this study, we considered a plate with finite width and length where tensile stress is applied. As the plate is subjected to remote tensile stress, lines of force are created along the length, and local stress is proportional to the density of lines of force. One case of our study with two cracks is shown in Fig.8, where lines of force are drawn schematically.

As stress cannot pass through a crack, lines of force are diverted around cracks which result in local stress concentration on cracks tip.

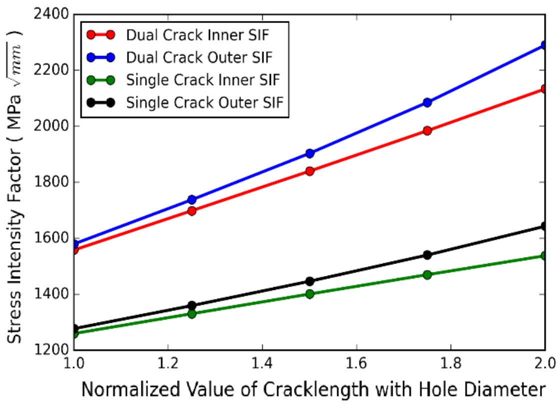


Fig. 7(a) Stress intensity factor vs. normalized value of crack length with hole diameter width for four different cracks tip using Displacement Extrapolation method.

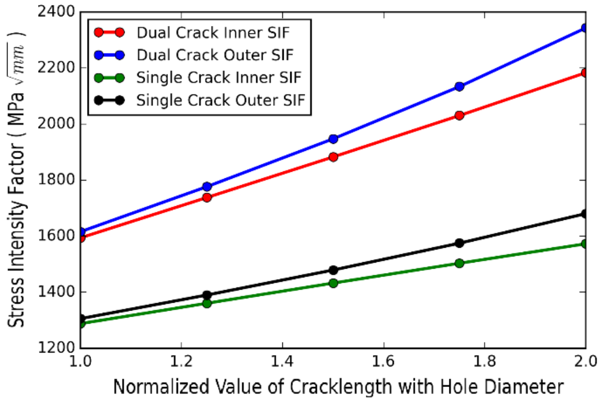


Fig. 7(b) Stress intensity factor vs. normalized value of crack length with hole diameter width for four different cracks tip using J-Integral method.

Table 2 Comparison of obtained results by Displacement Extrapolation method and J-integral method with the analytical solution.

Solution method	Dual crack	Error	Single crack	Error
Analytical	1228.1	-	1102.76	-
Displacement extrapolation	1260	2.6%	1115	1.11%
J-integral	1290	5.04%	1142	3.55%

Now when two cracks are present in the plate, they reduce more space to transmit lines of force than a single crack, resulting in a higher stress intensity factor. When crack size increase, it reduces the space for lines of force, eventually stress intensity factor increases. This same argument validates the results for normalized eccentricity when the crack changes its position from center of the plate to edge of the plate. When a crack forms far from center, it stops the path for lines of force in a short region which creates a higher density of these force lines and increases the value of K_I [17].

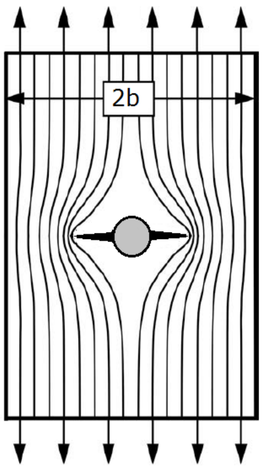


Fig. 8 Stress concentration effect due to dual crack with a circular hole in a finite plate.

6. CONCLUSIONS

Displacement extrapolation and J-integral methods are employed to calculate the stress intensity factor in this study. Stress intensity factor for four different crack tips with a circular hole is evaluated with 1 mm 8-node quadratic element. Effect of crack length is studied along with the effect of cracks position. Following conclusions are drawn:

- Obtained results agree with the analytical solution within acceptable accuracy for both displacement extrapolation and J-integral methods although the accuracy of SIF calculation is affected by mode of solving method and crack tip types.
- In the case of variable eccentricity, SIF values remain constant for eccentricity values from 0 to 0.465 and then follow a parabolic trend. This behavior is observed for all four crack tips.
- SIF values for different crack lengths show an increasing linear relation with normalized crack lengths.

In a nutshell, it can be said that a high eccentricity of the hole and a large crack length represents a most dangerous crack that is more likely to fail.

7. ACKNOWLODGEMENT

Authors of this article would like to thank Multiscale Mechanical Modeling and Research Networks (MMMRN) for their guidelines and comments to make this investigation more enriched.

REFERENCES

[1] P. Paris and F. Erdogan, “A critical analysis of crack growth laws”, J Basic Eng. Trans., ASME, Vol. 5, pp. 528–34, 1963.

[2] Y. Murakami *et al*., Stress intensity factors handbook, Pergamon Press, 1987.

[3] H. Tada, P. Paris, G. Irwin, The stress analysis of cracks handbook, Paris Productions Inc. 1985.

[4] R. Liu, H. Huang, J. J. Liu, W. Wang, L. Rong and Z. R. Nie, “Finite element analysis on the effect of the texture to the crack tip plasticity in aluminum alloy”, Materials Science Forum, Vol. 850, pp. 328, 2016.

[5] P. Weißgraeber, J. Felger, D. Geipel and W. Becker “Cracks at elliptical holes : Stress intensity factor And Finite Fracture Mechanics solution”, European Journal of Mechanics - A/Solids, Vol.55, pp. 192–198, 2016.

[6] Q. Han, Y. Wang, Y. Yin and D. Wang, “Determination of stress intensity factor for mode I fatigue crack based on finite element analysis”, Engineering Fracture Mechanics, Vol. 138, pp. 118–126, 2015.

[7] F. Shang, M. Kuna and M. Abendroth, “Finite element analyses of three-dimensional crack problems in piezoelectric structures”, Engineering Fracture Mechanics, Vol. 70, pp. 143–160, 2003.

[8] K. Y. Lin and J. W. Mar, “Finite element analysis of stress intensity factors for cracks at a bi-material interface”, International Journal of Fracture, Vol. 12, No.4, 1976.

[9] P. Noury and K. Eriksson, "Determination of stress intensity factors for cracked bridge roller bearings using finite element analyses", Engineering Fracture Mechanics, Vol. 169, pp. 67–73, 2017.

[10] W. X. Zhu and D. J. Smith, "On the use of displacement extrapolation to obtain crack tip singular stresses and stress intensity factors", Engng Fract Mech, Vol. 21(3), pp. 391–400, 1995.

[11] L. J. Kirthan, R. Hegde, V. A. Girisha and R. G. Kumar, "Evaluation of mode 1 stress intensity factor for edge crack using displacement extrapolation method", International Journal of Materials and Structural Integrity, Vol. 10, no. 1–3, pp. 11–22, 2016.

[12] J. R. Rice, "A path independent integral and the Approximate analysis of strain concentrations by notches and cracks", J. Appl. Mech., Vol. 35, pp. 376–86, 1968.

[13] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J. Ghazali, "Finite element analysis of J-Integral for surface cracks in round bars under combined mode I loading", International Journal of Integrated Engineering, Vol. 9, no. 2, 2017.

[14] S. K. Chan, L. S. Tuba and W. K. Wilson, "On the finite element method in linear fracture mechanics", Engineering Fracture Mechanics, Vol. 2, pp.1–17, 1970.

[15] ANSYS®, Release 16.2 - © SAS IP, Inc.2016.

[16] ANSYS Mechanical APDL Theory Reference. Release 16.2. ANSYS, Inc. 2016.

[17] T. L. Anderson, Fracture Mechanics Fundamentals and Applications, Taylor & Francis, 2005.