Published Online March 2019 (http://www.cuet.ac.bd/merj/index.html)



**Mechanical Engineering Research Journal** 

Vol. 11, pp. 66–71, 2018



# MATHEMATICAL SIMULATION OF DRIVER SEAT SUSPENSION SYSTEM USING QUASI-ZERO STIFFNESS SYSTEM

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**Abstract:** Machine vibration is unwanted and unanticipated. Main purpose of vehicle seat suspension system is to isolate vibration at its lowest possible level. Human body is least tolerant of vibration in excitation frequency range of 4 to 8 Hz. The transmissibility between chassis and seat is about 4 to 8 Hz. This study represents passive Quasi-Zero Stiffness (QZS) seat suspension system which combines a positive stiffness element and negative stiffness element, results in high static stiffness to mount weight of payload and low dynamic stiffness to increase vibration isolation capability. Potential energy method is used to obtain mathematical formulation of spring-mass vibration isolation system (NSS) is 1.3 Hz. It is noted that natural frequency can be reduced to 0.65 Hz by using two NSS which is lower than conventional vehicle seat suspension system. Along with that natural frequency, transmissibility of a seat suspension system can be reduced by great extent using two NSS. This paper has been representing a mathematical simulation of vehicle drivers' seat suspension system using QZS for riding and driving safety.

Keywords: Vibration, Quasi-Zero Stiffness, Spring, Potential Energy Method, Natural Frequency.

# 1. INTRODUCTION

A vehicle is usually specified by engine horse power, torque and zero to hundred kilometers per hour speed or above [1]. However, horse power propagated by vehicle engine will be worthless if vehicle does not offer proper suspension system or vehicle drivers cannot control the vehicle properly. Suspension system is one of the most important considerations in designing a vehicle [2]. It is noted that suspension of vehicle performs multiple function such as retaining contact between vehicles tires and road surface, keeping stability of vehicle, and can isolate the frame of vehicle from road- induced vibration and shocks [3].

Seat is an important part of automobile and performs an effective role to attenuate vehicle vibration, has an effect on the comfort ride. Passenger's comfort is one of the important considerations in designing a suspension system at the same time it is very important to think about drivers' seat suspension system. Vehicle drivers' comfort and proper control over vehicle have been extensively studied and considered with great importance at the present time [1-3]. Vibration creates motion of a body or particle or a system connected with another bodies displaced from a position of equilibrium [4].

Most vibrations are unwanted not only in human body but also in machines and structures because they produce increased stress, energy losses, causes added wear, increase bearing loads, induce fatigue, create discomfort and fatigue of vehicle drivers, passengers and absorb energy from the system. Along with that vibration can be harmful, very risky, and uncomfortable for vehicle drivers in riding.

Fig. 1 shows that human body is least tolerant of the vertical vibration in the excitation frequency range between 4 to 8 Hz. The transmissibility between chassis and seat is in the range of 4 to 8 Hz, which is still large. The transmissibility between chassis and seat can be reduced by reducing the natural frequency of vehicle suspension system.





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It is worthy to mention that a total of 1,422 people were killed and 1,289 others injured in 1,489 road accidents in 2016 in Bangladesh along with that at least 4284 people were killed and 9112 others injured in 3472 road accidents in 2017 in Bangladesh [5]. The number of accidents is increased by 25.56% and death tolled by 50.63% in 2017 compared to that of 2016. The National Committee to Protect Shipping, Roads and Railways (NCPSRR) mentioned seven vital reasons for accidents and one of them is long-term driving with fatigueness created due to whole-body vibration. It is noted that long-term driving leads to low back pain which is common muscles skeletal impairments [6]. The spine lapses its fixity when the muscles and ligaments become weak [7].Mechanical vibrations in the infra-frequency range affect comfort, health and safety problems. Hence, vehicle seat design with lower natural frequency and high load bearing capacity is important factor in reducing back pain.

Table 1 shows road accidents and casualties statistics years (2009-2017) in Bangladesh

Table 1 Road accident and casualties statistics Years (2009-2017) in Bangladesh Courtesy of data of BRTA  $\left[5\right]$ 

Year	Number of Accidents	Death	Injury
2009	3381	2958	2686
2010	2827	2646	1803
2011	2667	2546	1641
2012	2636	2538	2134
2013	2029	1957	1396
2014	2027	2067	1535
2015	2394	2376	1958
2016	2978	2844	2320
2017	3742	4284	9112

Recently, an alarming rise in road accidents, significantly highway accidents in Bangladesh is observed nowadays. If vehicle drivers don't feel comfort due to severe vibration while riding, he/she cannot have proper control over vehicle. This type of driving creates fatigue to the drivers and causes many accidents, even leads to deaths. It is noted that vibration level transferred from chassis to drivers' seat can be controlled or reduced when drivers' discomfortness is reduced. Hence, the vehicle drivers can have proper control in riding and chance of road accident will be reduced.

Designing vehicle drivers' seat with proper suspension system can reduce drivers' fatigueness created while driving and can reduce the number of road accidents happened due to losing control over vehicle for vibrational problem in improper road profile. Vehicle drivers' seat can be modeled and analyzed using Quasi-Zero Stiffness system. Vehicle seat suspension with passive Quasi-Zero Stiffness system provides large vibration isolation bandwidth. Quasi-Zero Stiffness system configures a positive stiffness element and a negative stiffness element in isolator [8]. Ideally, positive stiffness should be equal to negative stiffness at an equilibrium position under designated load, which results in definition of an isolator with high static stiffness and low dynamic stiffness called Quasi-Zero Stiffness system [9]. It is noted that QZS has been used to design vehicles drivers' new seat suspension system using springs, dampers, magnets and others mechanical configurations [6–11]. This paper represents a mathematical simulation of vehicles driver seat suspension system using Quasi-Zero Stiffness for riding and driving safety.

# 2. MATHEMATICAL MODEL OF NEW SEAT SUSPENSION SYSTEM

Vibration is transmitted through transmission path to receiver from source of vibration. Transmission path is main point where different modification can be performed to reduce vibration transmission. NSS system has been added in transmission path to design a vehicle drivers' seat suspension system to reduce vibration level. Conventional seat suspension system consists of a payload mass of (M) supported by linear spring of stiffness (K). Effective vibration isolation occurs when excitation frequency becomes equal to

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$
(1)

The (Eq. 1) implies that decreasing stiffness of spring leads to decrease in excitation natural frequency range or bandwidth and leads to an increase of static deflection of mass. On the other hand, increasing of payload mass decreases natural frequency but this is not desirable because of causing stability problem. Passive QZS vibration isolation system reduces natural frequency by minimizing stiffness of a system.



Fig. 2 Un-damped passive vibration isolation system with two sided NSS.

Where, L = Length of bar, X = Displacement of mass of system, Y = Input displacement,  $\partial_0$  = Initial deflection of spring, K = Stiffness of main spring, k<sub>1</sub> = Stiffness of tension spring.

Potential energy of the system as shown in Fig. 2 without NSS is as follows

$$U_1 = \frac{1}{2}KX^2 \tag{2}$$

Potential energy of the added NSS system is as follows

$$U_{2} = \frac{1}{2} \times 2_{k1} (\partial_{O-L+} \sqrt{L^{2} - X^{2}})^{2}$$
$$= k_{1} (\partial_{O-L+} \sqrt{L^{2} - X^{2}})^{2} \qquad (3)$$

Where,  $\partial_o$  is the initial deflection of negative spring from equilibrium position. Hence, Total potential energy of the system can be reduced to

$$U(x) = \frac{1}{2} K X^{2} + {}_{k1} (\partial_{o-L} + \sqrt{L^{2} - X^{2}})^{2}$$
(4)

It is noted that 1<sup>st</sup> derivative of potential energy function of an elastic system with respect to displacement is spring force [4]. From (Eq. 4), we get,

$$\frac{\partial U}{\partial x} = KX - 2k_1 \left\{ 1 + \frac{\partial_0 - L}{\sqrt{L^2 - X^2}} \right\}_X$$
(5)

In addition, the 2<sup>nd</sup> derivative of potential energy function of an elastic system with respect to displacement is stiffness of the elastic system [4]. Hence, (Eq. 5) is reduced to

$$\frac{\partial^2 U}{\partial X^2} = \mathbf{K} - 2\mathbf{k}_1 - \frac{\partial^2}{\partial x^2} \left\{ \frac{(\partial_0 - \mathbf{L}) x}{\sqrt{L^2 - X^2}} \right\}$$
(6)

However, the system will be linear and natural frequency of linear system will be lower than the natural frequency of nonlinear system at  $\partial_0 = L$  [4]. Hence, from (Eq. 4) the potential energy function can be written at  $\partial_0 = L$  as follows,

$$U(x) = \frac{1}{2} K X^{2} + k_{1} L^{2} - k_{1} X^{2}$$
(7)

From Eq. (5), spring force can be written at  $\partial_0$ =L as follows, Spring force,

$$F(X) = KX - 2k_1X \tag{8}$$

From Eq. (6), stiffness of the system can be written at  $\partial_0$ =L as follows,

$$\frac{\partial^2 U}{\partial X^2} = K_{\text{Total}} = K - 2k_1 \tag{9}$$

Hence, natural frequency of new proposed system can be written as,

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K_{Total}}{M}}$$
(10)

Transmissibility of this system can be written as if a damper is used [4],

$$\frac{X}{Y} = \sqrt{\frac{4\epsilon^2 \left(\frac{f}{fn}\right) 2 + 1}{\left\{1 - \left(\frac{f}{fn}\right) 2 + 4\epsilon^2 \left(\frac{f}{fn}\right) 2\right\}}}$$
(11)

Similarly, phase angle can be written as [4],

$$\emptyset = \tan^{-1} \left[ \frac{-2\varepsilon \left(\frac{f}{f_n}\right)3}{1 - \left(\frac{f}{f_n}\right)2 + 4\varepsilon^2 \left(\frac{f}{f_n}\right)2} \right]$$
(12)

Where, zeta is a term related to damping co-efficient, f is system's exciting frequency and  $f_n$  is the system's natural frequency.

# **3. SEAT SUSPENSION SYSTEM**

Seat suspension system can be grouped into three categories such as passive, semi-active and active suspension system. Passive seat suspension system is most commonly used

in vehicle because of its low cost and having good vibration isolation bandwidth. Semi-active and active suspension systems having good vibration isolation bandwidth are often used in vehicles because of their high cost. Passive suspension system is also termed as spring-mass suspension system because of the only use of mass and spring.

#### Case 01: Seat Suspension System without QZS System

Fig. 3 shows old approach of seat suspension system. At the very beginning of vehicle invention, this concept was used in vehicle. Here, a main spring of stiffness (K) is used and payload of the system is M.



Fig. 3 Conventional seat suspension system without NSS.

Potential energy of this system can be written as

$$U = \frac{1}{2}KX^2$$
(13)

Natural frequency of this system is as follows

$$f_{n=\frac{1}{2\pi}\sqrt{\frac{K}{M}}}$$
(14)

The (Eq.14) shows that natural frequency can be reduced by reducing stiffness and increasing mass of the system. But it is not possible for stability concern. Quasi-Zero Stiffness system can be introduced for overcoming stability problem.

#### Case 02: Seat Suspension System with Single NSS

A main spring of stiffness (K) is used along with a NSS  $(k_1)$  as shown in fig. 4. Added negative stiffness is the easy and economical way to reduce stiffness of main system without changing its main vibration isolation bandwidth. It is aforementioned that natural frequency can be reduced by reducing main system's stiffness.



Fig. 4 Seat suspension system using main spring with single NSS.

Potential energy of this system can be written as

$$U = \frac{1}{2}KX^{2} + \frac{1}{2}K_{1}(\partial_{0-L^{+}}\sqrt{L^{2} - X^{2}})^{2}$$
(15)

Natural frequency of this system is as follows

$$f_{n=} \frac{1}{2\pi} \sqrt{\frac{K-k1}{M}} \tag{16}$$

#### Case 03: Seat Suspension System with Two NSS

A main spring of stiffness (K) is used along with two NSS  $(k_1)$  as shown in fig. 5. This system reduces stiffness of main spring two times than the system as shown in fig. 4. Natural frequency and transmissibility is reduced more in this system as compared to case 01 and case 02.



Fig. 5 Seat suspension system using main spring with two NSS.

Potential energy of this system can be written as

$$U(x) = \frac{1}{2} K X^{2+} k_1 (\partial_{0-L+} \sqrt{L^2 - X^2})^2$$
(17)

Natural frequency of this system can be written as

$$f_{n=} \frac{1}{2\pi} \sqrt{\frac{K-2k1}{M}} \tag{18}$$

Table 2 shows parameter values used in computer simulation. These values are used for getting simulation results for the above mentioned three cases.

Table 2	Parameter	values	used	in	computer	simu	lation
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SI No.	Name	Symbol	Unit
01	Mass	М	60 kg
02	Stiffness (Main Spring)	K	4000 N/m
03	Negative Stiffness(Added in both side)	$k_1$	1500 N/m
04	Bar Length	L	50mm
05	Initial Spring deflection	$\partial_{o}$	50mm

### 4. RESULTS AND DISCUSSION

First and foremost purpose of seat suspension system is to attenuate vibration to a lowest possible level. Approaches of seat

suspension system depending on arrangement of springs and other elements are analyzed in this research. Fig. 6 shows variation of natural frequency with respect to added negative stiffness. It is noted that lower the natural frequency of a system, the better vibration attenuating power of the system. Reduced natural frequency vibration isolation system is preferred in vehicles' seat suspension system for drivers' comfortness.

Fig. 6 shows that natural frequency of seat suspension system, where is no negative stiffness element used, only system's main spring is used, is obtained 1.3 Hz. Natural frequency of seat suspension system using single NSS, where is one negative stiffness element used, is obtained 1.02 Hz. It is noticeable that natural frequency of seat suspension system, where are two negative stiffness element used, is obtained 0.65 Hz. By using same parameter, natural frequency can be reduced by a significant amount in new seat suspension system.



Fig. 6 Schematic representation of natural frequency vs. added negative stiffness.

It is noted that if change of potential energy of a single degree of freedom system with respect to displacement is reduced, natural frequency of that system is reduced. Fig. 7 shows change of potential energy with respect to displacement. The change of slope of potential energy curve of new seat suspension with two NSS system is lower than that of seat suspension with single NSS system and original seat suspension system.

Figure 7 represents that **c** is the slope of potential energy curve of seat suspension system without NSS, **b** is the slope of potential energy curve of seat suspension system with single NSS, **a** is the slope of potential energy curve of seat suspension system with double NSS. It is very clear that c>b>a that means slope of potential energy curve of seat suspension system with double NSS is lower than slope of potential energy curve of seat suspension system with single NSS and slope of potential energy curve of seat suspension system with single NSS and slope of potential energy curve of seat suspension system with single NSS and slope of potential energy curve of seat suspension system without NSS



Fig. 7 Schematic representation of potential energy vs. displacement.

Fig. 8 shows transmissibility vs. input frequency graph for new suspension system with variable ( $\varepsilon$ ) if a damper is used in the system as shown in fig 5. Transmissibility is reduced with the increase of  $(\varepsilon)$ . A suspension system of lower transmissibility is preferable in vibrating environment.



Fig. 8 Schematic representation of transmissibility vs. input frequency of seat suspension with two NSS for variable zeta.

Figure 9 shows transmissibility vs. excitation input frequency graph for an un-damped passive new suspension system. From Fig. 9, it is seen than transmissibility of suspension system with double NSS is lower than system with single NSS and system without NSS. Low transmissibility means its output displacement is low. System with double NSS reaches at its natural frequency point when transmissibility value is 40, after that point its vibrating tendency is gradually reduced.



Fig. 9 Schematic representation of transmissibility vs. input frequency.

# 5. CONCLUSION

This paper represents a mathematical analysis of vehicles' driver seat suspension system using QZS for riding and driving safety. It is noted that natural frequency is obtained as 1.3 Hz without using any negative stiffness system, 1.02 Hz using single QZS system and 0.64 Hz using two-QZS system. It is worthy to mention that same parameter values are used for these three simulation cases. Natural frequency can be reduced to 0.64 Hz from 1.3 Hz using QZS system. In addition, transmissibility of seat suspension system with two QZS system is reduced about more than half compared with seat suspension system with single QZS system. This research work only uses commercial spring, mass, bar and mechanical bearing. It is noted that the cost of new vehicles' driver seat suspension system will be cheaper than that of active, semi-active, and pneumatic seat suspension system.

# 6. ACKNOWLEDGEMENTS

This research work is carried out at Department of Mechanical Engineering, CUET. The authors would like to

express their gratitude and thanks to Directorate of Research and Extension, CUET for giving financial help for this research.

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