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Optical plane waves solution with bifurcation behaviors and chaotic motion for resonant time-fractional nonlinear Schrödinger equation having Kerr law

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ABSTRACT

This study investigates the resonant optical oblique solitons with dynamical behaviors described by (2+1)-dimensional time-fractional nonlinear Schrödinger equation (TFNLSE) having Bohm's quantum potential and Kerr law nonlinearity. The conformable Khalil's fractional derivative with a traveling-wave transform is implemented to convert the TFNLSE into the nonlinear ordinary differential equation (NODE). The planar dynamical systems are also formed to study the considered equation's bifurcation behaviors and chaotic motion. The modified simple equation method (MSEM) is used to divulge the optical plane-wave solution of FNLSE. The effects of obliqueness and fractional parameters on obtained results are demonstrated graphically along with physical descriptions. Results of the study revealed that the nonlinear wave phenomena and dynamical properties are changed following the increase of obliqueness and fractional parameters. The outcomes would be beneficial for better understanding the basic features of bifurcation properties and chaotic motion of resonant optical solitons in nonlinear optics, specifically in optical bullets, Madelung fluids, etc.

Keywords: Optical soliton; Resonant nonlinear Schrödinger equation; Bifurcation; Chaoticmotion; The modified simple equation method.

1. Introduction

It is well known that electromagnetic waves are produced in a physical situation, such as plasmas, microwaves, photonic metamaterials, optical bullets, etc. With the static ambient magnetic field. That is, two directions (parallel and perpendicular) of the oscillatory wave to the magnetic field are only countable to investigate wave phenomena in such situations. It is provided that the perturbations may be assumed to change and spread inxz-plane, without loss of generality. Not only in such but also in many physical conditions, the obliqueness may not be ignored. It is challenging to examine the oblique plane wave because plane waves are not basically incident. At these stages, the underlying issues of a plane wave brothering along a single axis that is randomly comparative to a rectangular coordinate system may be considered. In order to overcome such difficulty, the direction cosines of the uniform plane wave may be included based on the equi-phase surfaces that may be assumed as planes at right angles to the direction of the surface. For instance, one can write [1] as $\vec{k}.\vec{r} = k(x\cos\vartheta + z\sin\vartheta)$ at the point (x,z) in the xz -plane, where $k$is constant, ϑ indicates the angle between two directions of propagation and$\cos^2\vartheta + \sin^2\vartheta = 1$. In most of the previous studies [2-10], researchers have obtained the wave solutions of nonlinear evolution equations (NLEEs) without considering obliqueness. It is, therefore, essential to divulge the analytical solutions with obliqueness by considering the two-dimensional NLEEs.

For this reason, the following (2+1)-dimensional TFNLSE having Bohm's quantum potential are chosen [11].

$$iD_t^\lambda u + \nabla^2 u + \sigma f(s)|u|^2 u + s\delta\left\{\frac{\nabla^2|u|}{|u|}\right\}u = 0, \quad (1)$$

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where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, $u(x,z,t)$ are measured the complex-valued wave function with $i = \sqrt{-1}$, D_t^λ is denoted the conformable derivative (CD) of $\lambda\,(0 < \lambda < 1)$ order, σ is constant-coefficient, and $f(s)$ is measured the types of nonlinearity, respectively. By including the non-linear Kerr - law, the Eq1.(1) is turning to

$$iD_t^\lambda u + \eta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)u + \sigma|u|^2 u + \delta\left\{\frac{1}{|u|}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)|u|\right\}u = 0, \tag{2}$$

The η and δ are expressing the speed coefficientparameters in the Kerr-Law nonlinearity. It is to be noted that Eq. (2) is applicable to understand the physical behaviors of plane wave optical solitons with obliqueness in nonlinear optics, optical bullet, photonic metamaterials, optical fiber communications, photonic bandgap, mitigate Internet bottleneck, collisionless plasmas as well as in diverse physical system having Madelung fluids [12-16], where the non-locality plays an essential role. Thus, the modified simple equation method (MSEM) [17, 18] is emerging to report for finding optical plane wave solutions with obliqueness of Eq. (2) based on the condition by forming the planar dynamical system (PDS). The chaotic motion of such PDS is also investigated.

2. Nonlinear ODEs with planar dynamical systems

let us start with the wave variable transform as
$u = e^{i\Lambda}U(\xi)$, (3) and

$$\left.\begin{aligned} \xi &= x\cos\vartheta + z\sin\vartheta + c\left(\frac{t^\lambda}{\lambda}\right) \\ \Lambda &= k(x\cos\vartheta + z\sin\vartheta) + \omega\left(\frac{t^\lambda}{\lambda}\right) \\ \cos^2\vartheta &+ \sin^2\vartheta = 1 \end{aligned}\right\}, \tag{4}$$

The parameters c, k, and ω express the reference frame speed, number of the wave, and angular frequency. It efficiently reduces the conformable fractional RNLSE into an ordinary differential equation of integer order regarding the transformations specified in Eq. (4). With the assistant of the property $D_t^\lambda(\tau^\gamma) = \gamma\tau^{\gamma-\lambda}$ For all $\gamma \in \Re$, of the conformable derivatives [19]. Hence, Eq. (2) is reduced to

$$(\eta+\delta)U''(\xi) - (\omega+\eta k^2)U(\xi) + \sigma U^3(\xi) = 0 \tag{5}$$

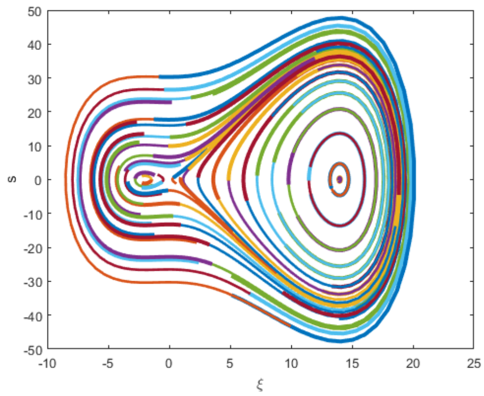
Although, the chaos is continuing and obstinate to determine long-term evolution under some particular mathematical condition. In this approach, the determinant system continues the nonlinearity [20]. Beforehand articles [20, 21] represented the integrability of equations evolution by applying exterior-perturbations, which had well-defined explanations. Moreover, the method of period-doubling, together with the quasi period-doubling, is required to withhold in the exterior-perturbations. Eq2 is rearranging to $(\eta+\delta)U''(\xi) - (\omega+\eta k^2)U(\xi) + \sigma U^3(\xi) = C_0\cos(k\xi)$. (6)

Where $C_0\cos(k\xi)$ shows the external periodic forcing term and C_0 along with k demonstrates the real constant.

Eq. (6) can be specified in the way of the PDS of three dynamical systems;

$$\left.\begin{aligned} U'(\xi) &= S \\ S'(\xi) &= \frac{(\omega+\eta k^2)}{(\eta+\delta)}U(\xi) - \frac{\sigma}{(\eta+\delta)}U^3(\xi) + \frac{C_0}{(\eta+\delta)}\sin(T) \end{aligned}\right\} \tag{7}$$

It is mentioned here that the existence of the external periodic term $C_0/(\eta+\delta)\sin(T)$ is affected, and is actually turned to FRNLSEs in the area of the chaotic-motion. To get the equilibrium points from Eq. (7), one can assume $U' = 0, S' = 0$ yields $(0,\ 0)$, $(\pm\sqrt{\rho},\ 0)$ with $\rho = (\omega+\eta k^2)/\sigma$. It is noted that three equilibrium points have existed for obtaining PDS. Based on the equilibrium points, Figure 1(a) and 1(b) shows the phase portrait and its vector field by choosing $\omega = 0.1,\ \eta = -3.5, k = 2, \sigma = -0.5, \delta = 0.5, \lambda = 0.5, C_0 = 0$ and $\rho = 1.5$. It is predicted from Fig. 1 that, $(0,\ 0)$, it is the saddle-node bifurcation of unstable mode and $(\pm\sqrt{\rho},\ 0)$is the center node bifurcation of stable way. There is a produce homoclinic orbit and numerous periodic orbits around the center points, indicating that the traveling wave propagation supports the considered equation to understate the physical issues in the relevant field. Figures 2 (a) and (b) display the chaotic motion and its vector field with external periodic force terms and the constant values of remaining parameters. It is highly found from Fig.2 that a strong chaotic motion is produced due to the influence of external periodic force term, which is surprisingly changed the dynamical behaviors of physical issues described by conformable fractional RNLSE.



1 (a)

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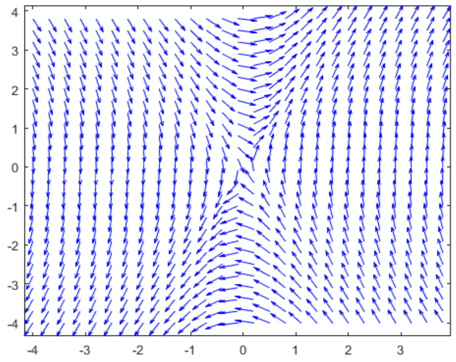
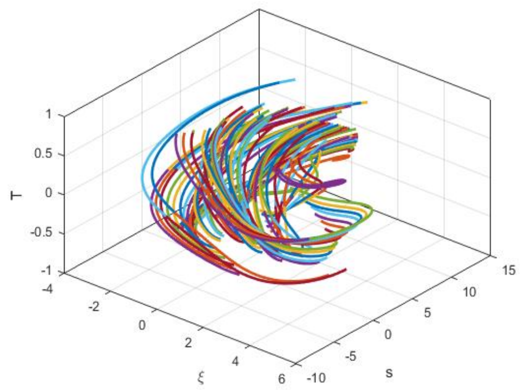


1(b)

Fig. 1 (a): Phase portrait and (b) its vector field of PDS with the constant values of $\omega = 0.1,\ \eta = -3.5, k = 2, \sigma = -0.5, \delta = 0.5, \lambda = 0.5, C_0 = 0$ and $\rho = 1.5$.



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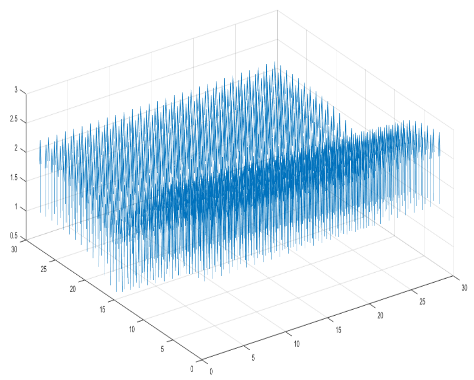

2(b)

Fig. 2 (a): Chaotic motion and (b) its vector field of PDS with the constant values of $\omega = 0.1,\ \eta = 0.001, k = 1, \sigma = 0.1, \delta = 0.1, \lambda = 0.5, C_0 = 0.5$ and $\rho = 0.5$.

3. Oblique plane wave solution via Modified simple equation method (MSEM)

Based on the MSEM [17, 18], the analytical solutions of Eq. (5) can be written as

$$U(\xi) = \sum_{k=0}^{N} \zeta_k \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^k , \tag{8}$$

Where ζ_k is an arbitrary constant, when $k = 0,1,2,3,\ \cdots\cdots$ and $\psi(\xi)$ is an unknown function for such values of $\zeta_k \neq 0$. It is noted that N it should be chosen by balancing the highest order of derivatives to the highest order of nonlinear terms that are involved in Eq. (5) and yields $N = 1$. From Eq.(8), one obtains

$$U(\xi) = \sum_{k=0}^{1} \zeta_k \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^k = \zeta_0 + \zeta_1 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right), \tag{9}$$

Where $\zeta_1 \neq 0$. Eq. (9) yields

$$U'(\xi) = \zeta_1 \left(\frac{\psi''(\xi)}{\psi(\xi)} - \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2\right), \tag{10}$$

$$U''(\xi) = \left(\zeta_1 \left(\frac{\psi'''(\xi)}{\psi(\xi)}\right) - 3\zeta_1 \left(\frac{\psi''(\xi)\psi'(\xi)}{\psi^2(\xi)}\right) + 2\zeta_1 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^3\right), \tag{11}$$

$$U^2(\xi) = \zeta_0^2 + 2\zeta_0\zeta_1 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + \zeta_1^2 \left(\frac{\psi'^2(\xi)}{\psi^2(\xi)}\right), \tag{12}$$

$$U^3(\xi) = \zeta_1^3 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^3 + 3\zeta_0\zeta_1^2 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2 + 3\zeta_1\zeta_0^2 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + \zeta_0^3, \tag{13}$$

By inserting Eq.(10)-(14) in Eq. (5) yields

$$(\eta^2 + \delta)\left(\zeta_1 \left(\frac{\psi'''(\xi)}{\psi(\xi)}\right) - 3\zeta_1 + 2\zeta_1 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^3\right) - (\omega + \eta k^2) + \sigma\left(\zeta_1^3 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^3 + 3\zeta_0\zeta_1^2 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2 + 3\zeta_1\zeta_0^2 \left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + \zeta_0^3\right) = 0.$$

Based on the coefficients of $(\psi(\xi))^0$,$(\psi(\xi))^{-1}$, $(\psi(\xi))^{-2}$ and $(\psi(\xi))^{-3}$,

the following equations are archived:

$$-(\omega + \eta k^2)\zeta_0 + \sigma\zeta_0^3 = 0, \tag{14}$$

$$(\eta^2 + \delta)\psi'''(\xi) + (-(\omega + \eta k^2) + 3\sigma\zeta_0^2)\psi'(\xi) = 0, \tag{15}$$

$$-(\eta^2 + \delta)\psi''(\xi) + (3\sigma\zeta_0\zeta_1)\psi'(\xi) = 0, \tag{16}$$

$$\zeta_1(2(\eta^2 + \delta) + \sigma\zeta_1^2)\psi'^3(\xi) = 0. \tag{17}$$

Now, the solution of Eq. (15) and Eq. (18) are obtained as

$$\zeta_0 = 0, \quad or \quad \zeta_0 = \sqrt{\frac{(\omega + \eta k^2)}{\sigma}} \tag{18}$$

and

$$\zeta_1 = 0, \quad or \quad \zeta_1 = \pm i\sqrt{\frac{2(\eta^2 + \delta)}{\sigma}}. \tag{19}$$

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When $\zeta_1 \neq 0$ and $\zeta_0 = 0$, Eq. (16) is provided only a trivial solution. Hence $\zeta_0 = 0$ is not zero for obtining the solution of the considered equation. In addition, Eq. (15) and Eq. (16) yields ;

$$\frac{\psi'''(\xi)}{\psi''(\xi)} = \frac{(\omega+\eta k^2-3\sigma\zeta_0^2)}{3\sigma\zeta_0\zeta_1}. \qquad (20)$$

By integating Eq. (20) givess

$$\psi''(\xi) = \alpha\, exp(\rho\xi) \qquad (21)$$

where $\rho = \frac{(\omega+\eta k^2-3\sigma\zeta_0^2)}{3\sigma\zeta_0\zeta_1}$

By substituting Eq. (21) in Eq. (16), one gets

$$\psi'(\xi) = \frac{\eta^2+\delta}{3\sigma\zeta_0\zeta_1}.(\alpha\, exp(\rho\xi))\,, \qquad (22)$$

Finaly, integrating Eq. (22) gives

$$\psi(\xi) = \frac{(\eta^2+\delta)(\alpha\, exp\,(\rho\xi))+\beta(3\sigma\zeta_0\zeta_1\rho)}{3\sigma\zeta_0\zeta_1\rho}. \qquad (23)$$

where α and β are integrating constants.

Thus, the analytical solution of Eq. (5) is obtained with the help of Eqs. (3), (10), and (23) in the following form:

$$u = e^{i\left[k(x\cos\vartheta+z\sin\vartheta)+\omega\left(\frac{t^\lambda}{\lambda}\right)\right]}\left\{\zeta_0 + \zeta_1\left[\frac{(\eta^2+\delta)\rho}{(\eta^2+\delta)(\alpha\, exp\,(\rho\xi))+\beta(3\sigma\zeta_0\zeta_1\rho)}(\alpha\, exp(\rho\xi))\right]\right\}, \qquad (24)$$

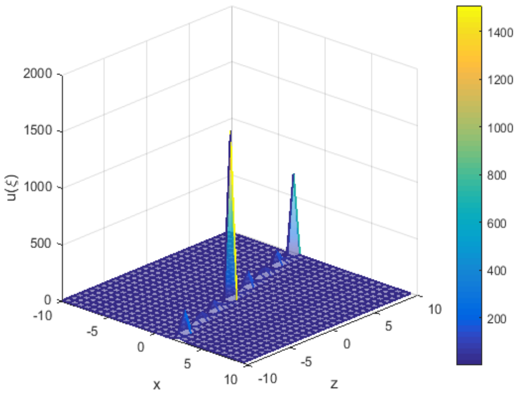
With $\xi = (x\cos\vartheta + z\sin\vartheta) + c\left(\frac{t^\lambda}{\lambda}\right)$ and

$\rho = (\omega + \eta k^2 - 3\sigma\zeta_0^2)/3\sigma\zeta_0\zeta$.

Based on the obtained solution of conformable fractional RNLSE, some of the outcomes are demo nested graphically in Figures 3, 4, and 5 with the constant values of the related parameters. It is found in Figs. 3-4 that the width and amplitude of traveling waves are changed with the change of obliqueness. In addition, the width and amplitude of traveling waves are slightly increased with the increase of fractional parameters.

4. Conclusions

In this work, the PDS has been formed from the considered RNLSE with conformable fractional time evolution for understanding the obliquely propagating physical issues of a plane wave in nonlinear physical systems. The dynamical behaviors of PDS have been investigated. It is observed that the considered equation has been supported the traveling wave phenomena. In addition, the solid chaotic motion has been occurred due to the inclusion of external periodic force term. The MSEM is then implemented to obtained oblique plane wave solutions from fractional RNLSE. Some of the plane waves have presented graphically, which indicates that obliqueness significantly changed the structures of such wave phenomena. Consequently, the result would be beneficial for better understanding the oblique plane-wave with their dynamical characteristics in nonlinear optics and many branches of physics



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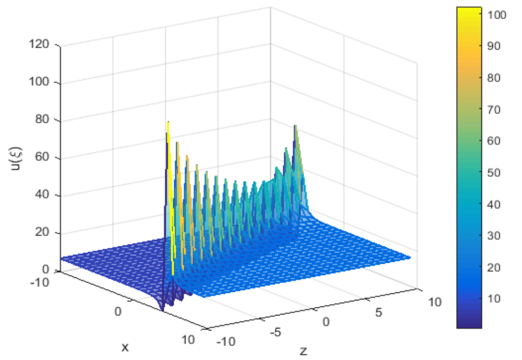

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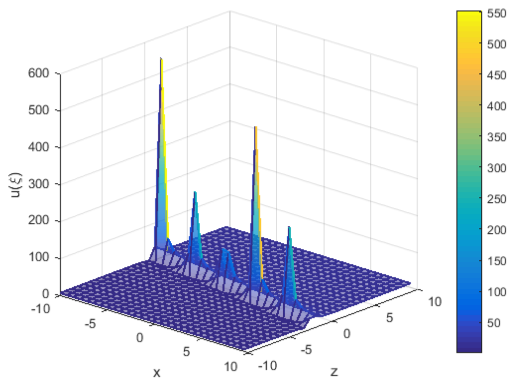

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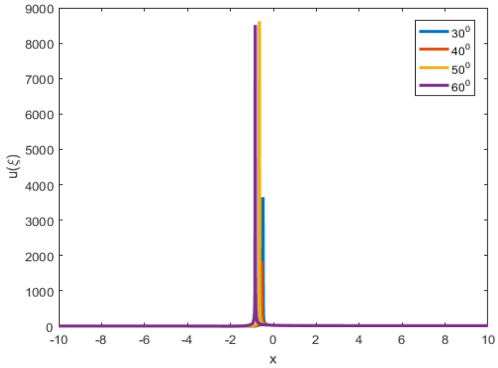
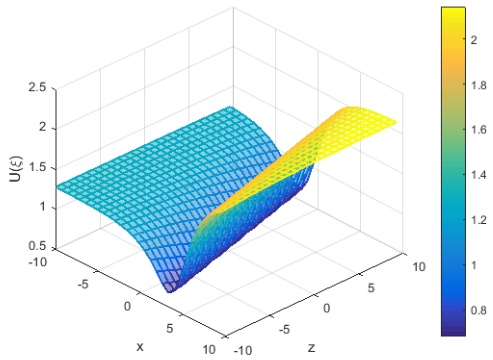


(d)

Fig. 3: Shape of the oblique plane wave $u(x,z,t)$ with regards to (a) x and $z$with $\vartheta = 15^0$, (b) x and $z$with $\vartheta = 27^0$, (c) x and z with $\vartheta = 80^0$ and (d) x keeping z as constant and different values of ϑ, respectively. The remaining parameters are chosen as $\omega = 0.1$, $\eta = -3.5$, $k = 2$, $\sigma = -0.5$, $c = 0.5$, $\delta = 0.5$, $\lambda = 0.5$, $\rho = 1.5$, $\beta = 0.2$, $\alpha = 0.3$ and $t = 2$.



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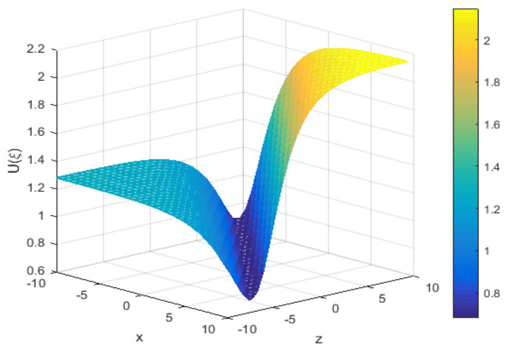

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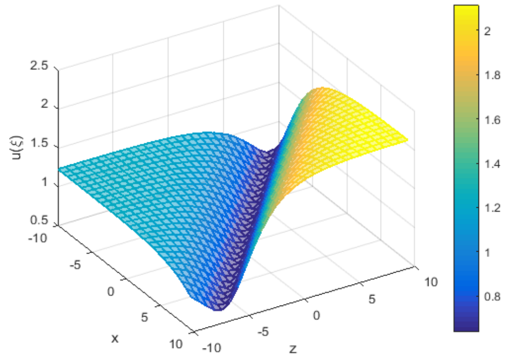

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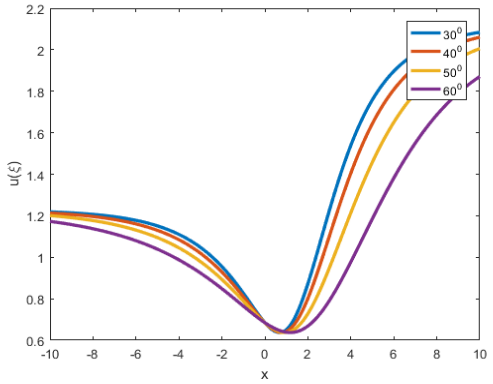

(d)

Fig. 4: Shape of the oblique plane wave $u(x,z,t)$ with regards to (a) x and z with $\vartheta = 15^0$, (b) x and z with $\vartheta = 50^0$, (c) x and z with $\vartheta = 80^0$ and (d) x keeping z as constant and different values of ϑ, respectively. The remaining parameters are chosen as $\omega = 0.1$, $\eta = 0.001$, $k = 1$, $\sigma = 0.1$, $c = 0.5$, $\delta = 0.1$, $\lambda = 0.5$, $\rho = 0.5$, $\beta = -0.2$, $\alpha = 0.1$ and $t = 1$.

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