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Theoretical Analysis of Non-Relativistic First Order Born Approximation For Ionization of H(2P) by Electron Impact

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Abstract

First Born triple differential cross sections (TDCS) has been calculated for the ionization of H(2P) by electrons in the coplanar asymmetric geometry following a multiple scattering theory. The present results show good qualitative agreement with the results of hydrogenic metastable 2S-state by electrons. The present reported work will be added in the experimental study for ionization of hydrogen atoms from metastable states..

Keywords: Ionization; Scattering; Cross-Section; Electron.
PACS number: 34.80Dp

1. Introduction

Atomic ionization of atoms by charged particles like electrons or positrons forms an important class in Atomic Physics. Accurate determination of cross-section results of various types such as single, double and triple differential with varied kinematic conditions [1-11] are important problems to study in Applied Mathematics. With the availability of new experimental results over the last four decades a new dimension is added in this field of research such as astrophysics, plasma physics and fusion research technology. It now appears that the investigation of ionization from metastable 2S-state [12-14, 20] and linear superposition of 2S and 2P state [21-22] of hydrogen atoms by charged particles is equally interesting. At present, beyond the first Born results, there exists the second Born calculation [12,21] for small momentum transfer at intermediate incident energies 150eV, 250eV and 400eV. Here we have presented a few new sets of theoretical results following the multiple scattering theory [9]. It may be noted that using this multiple scattering theory [9], cross sections show good results for the electron-atom ionization both in the ground state [9-12] and the metastable 2S state [13-15]. The energy spectrum of scattered electrons in the K-shell ionization of medium-heavy atoms by fast electrons was nicely represented [16-19] by applying the scattering state wave function of Das and Seal [1, 9] multiplied by a suitable spinor. For the ionization of hydrogen atoms by electrons from the metastable 2S and 2P states, there is no such TDCS study in literature, both theoretically and experimentally. So far we know, the present reported work is the first TDCS treatment for the ionization of metastable 2P-state hydrogen atoms by electrons both theoretically and computationally. Therefore, the present first Born results for ionization of hydrogen atoms by electrons in metastable 2P state are expected to be good, interesting and significant in the respective field of research.

2. Theory

The most detailed information presently available about single ionization processes of the following type

$$e^- + H(i) \rightarrow H^+ + 2e^- \qquad (1)$$

where, the symbol i denotes the initial state of the target, has been obtained in the coplanar geometry by analyzing triple differential cross sections (TDCS) measured in (e,2e) coincidence experiments. TDCS is a measure of the probability that in an (e,2e) reaction an incident electron of momentum $\overline{p}_i$ and energy E_i will produce on collision with the target two electrons having energies E_1 and E_2 and momenta $\overline{p}_1$ and $\overline{p}_2$ emitted respectively into the solid angles $d\Omega_1$ and $d\Omega_2$ centered about the directions (θ_1, ϕ_1) and (θ_2, ϕ_2). The TDCS is usually denoted by the symbol $d^3\sigma / d\Omega_1 d\Omega_2 dE_1$. For unpolarized incident electrons and targets, it is a function of the quantities

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E_i, E_1 or E_2, θ_1, θ_2 and $\phi = \phi_1 - \phi_2$. By integrating the TDCS over $d\Omega_1$, $d\Omega_2$ or dE_1, one can form various double and single differential cross sections. Finally, the total ionization cross section is obtained by integrating over all outgoing scattering angles and energies, and depends only on E_i, the incident electron energy. It is useful when studying (e, 2e) coincidence experiments to distinguish between several kinematical arrangements, since these have important implications for the theoretical analysis of the collision. A distinction can be made between coplanar and non-coplanar geometries. In coplanar geometries, the momenta $\bar{p}_i$, $\bar{p}_1$ and $\bar{p}_2$ are in the same plane and in non-coplanar geometries, the momentum $\bar{p}_2$ is out of the $(\bar{p}_i, \bar{p}_1)$ reference plane.

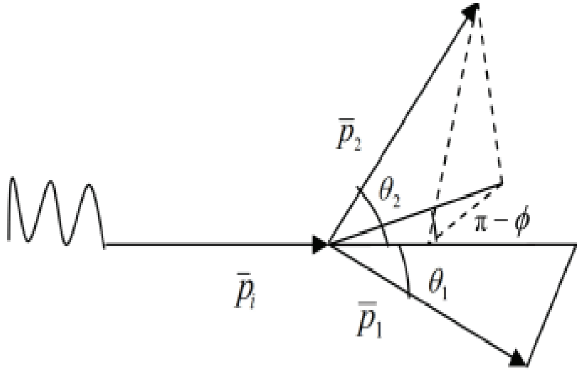

Figure. : The kinematics of an (e,2e) reaction. The incident electron momentum is $\bar{p}_i$ and the momenta of the outgoing electrons are $\bar{p}_1, \bar{p}_2$, respectively. Also the angles θ_1 and θ_2 are shown with respect to the incident direction and the angle $\pi - \phi$ are measuring the direction in coplanar situation.

Here we have described the multiple scattering theory [9] very briefly for the particular case of hydrogenic metastable 2P states at intermediate and high energies. The direct T-matrix element for ionization of hydrogen atoms by electrons may be written as

$$T_{fi} = \left\langle \Psi_f^{(-)}(\bar{r}_1, \bar{r}_2) \middle| V_i(\bar{r}_1, \bar{r}_2) \middle| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (2)$$

where the perturbation potential $V_i(\bar{r}_1, \bar{r}_2)$ is given by

$$V_i(\bar{r}_1, \bar{r}_2) = \frac{1}{r_{12}} - \frac{Z}{r_2}$$

For hydrogen atom, the nuclear charge is Z = 1, r_1 and r_2 are the distances of the two electrons from the nucleus and r_{12} is the distance between the two electrons.

The initial channel unperturbed wave function is

$$\Phi_i(\bar{r}_1, \bar{r}_2) = \frac{e^{i\bar{p}_i . \bar{r}_2}}{(2\pi)^{3/2}} \phi_{2p}(\bar{r}_1)$$

$$\Phi_i(\bar{r}_1, \bar{r}_2) = \frac{e^{i\bar{p}_i . \bar{r}_2}}{8\sqrt{2\pi^2}} r_1 \cos\theta e^{-r_1 \lambda_1} \qquad (3)$$

where,

$$\phi_{2p}(\bar{r}_1) = \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-r_1/2}$$

$$= \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-\lambda_1 r_1} \quad , \quad \left[\lambda_1 = \frac{1}{2}\right]$$

is the hydrogenic metastable 2P state wave function, $\bar{p}_i$ is the incident electron momentum, and $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is the final three-particle scattering state with the electrons being in the continuum with momenta $\bar{p}_1, \bar{p}_2$. Co-ordinates of the two electrons are taken to be $\bar{r}_1$ and $\bar{r}_2$. Here $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is approximated by a wave function [1] and is given by [9] as

$$\psi_f^{(-)}(\bar{r}_1, \bar{r}_2) = N(\bar{p}_1, \bar{p}_2) \Big| \phi_{\bar{p}_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2 . \bar{r}_2} + \phi_{\bar{p}_2}^{(-)}(\bar{r}_2) e^{i\bar{p}_1 . \bar{r}_1}$$

$$+ \phi_{\bar{p}}^{(-)}(\bar{r}) e^{i\bar{P} . \bar{R}} - 2e^{i\bar{p}_1 . \bar{r}_1 + i\bar{p}_2 . \bar{r}_2} \Big| / (2\pi)^3 \qquad (4)$$

where $\quad \bar{R} = \dfrac{\bar{r}_1 - \bar{r}_2}{2} \quad , \quad \bar{R} = \bar{r}_1 + \bar{r}_2 \quad ,$

$$\bar{p} = (\bar{p}_2 - \bar{p}_1), \quad \bar{P} = \bar{p}_2 + \bar{p}_1$$

The normalization constant $N(\bar{p}_1, \bar{p}_2)$ is given by

$$|N(\bar{p}_1, \bar{p}_2)|^{-2} = \left| 7 - 2[\lambda_1 + \lambda_2 + \lambda_3] - \left[\frac{2}{\lambda_1} + \frac{2}{\lambda_2} + \frac{2}{\lambda_3}\right] \right.$$

$$\left. + \left[\frac{\lambda_1}{\lambda_2} + \frac{\lambda_1}{\lambda_3} + \frac{\lambda_2}{\lambda_1} + \frac{\lambda_2}{\lambda_3} + \frac{\lambda_3}{\lambda_1} + \frac{\lambda_3}{\lambda_2}\right] \right| \qquad (5)$$

where

$$T_{\mathrm{B}} = \left\langle \phi^{(-)}_{\bar{p}_1}(\bar{r}_1) e^{i\bar{p}_2.\bar{r}_2} \middle| V_i \middle| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (7)$$

is the first order Born Approximation term.

The triple differential cross-section is finally calculated by

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{p_1 p_2}{p_i} \left| T_{fi} \right|^2 \qquad (8)$$

where E_1 is the energy of the ejected electron.

After analytical calculation, we have computed these expressions (8) numerically using computer programming language.

3. Results and Discussion

We have presented here the triple differential cross sections (TDCS) of the first Born approximation for the process (1) at incident energy $E_i = 250eV$ for some varied ejected angles and fixed scattering angles. The present results are displayed in six figures where we have plotted the electron impact TDCS varying against the angle of ejection (θ_1) of the ejected electron. In all figures, the region for $\theta_1(0^0 - 150^0)$ and $\phi = 0^0$, refers to the recoil region, while $\theta_1(150^0 - 360^0)$ and $\phi = \pi$, refers to the binary region. We consider $\theta = 0^0$ to $\theta = 360^0$ here.

The present first Born approximation TDCS results in H(2P) are qualitatively compared with hydrogenic 2S-state results [13] for scattering in a plane. Here the scattering angles are taken as $\theta_2 = 5^\circ$ (Fig. 1), 7° (Fig. 2), 9° (Fig. 3), 11° (Fig. 4), 15° (Fig. 5), 20° (Fig. 6) and the ejected angles θ_1 are varied from 0^0 to 360^0 (measured oppositely from the forward direction compared to that for scattering angle θ_2).

Here we notice that the present first Born TDCS results (Figures 1-6) for ionization of metastable 2P-state by electrons exhibit two distinct peaks both in the recoil and the binary regions at the incident energy $E_i = 250eV$. The binary peak arises due to the electron-electron interaction while the recoil peak is due to the projectile nucleus interaction.

We remark that the present study results at incident energy $E_i = 250eV$ and the higher scattering angles show peaks with dissimilar magnitude in the binary region. All of the figures reveal that at incident energy $E_i = 250eV$ and at higher scattering angles, the collision provides less prominent recoil peaks in metastable 2S-state [13] than the present metastable 2P-state results. It may be expected. On the other hand, in the binary region, the present peak values are slightly smaller than the corresponding compared hydrogenic 2S-state results [13]. This may be happen because of the change of states.

The present binary peaks are shifted to higher ejected angles in the recoil region (Figures 1-6) whereas in the binary regions, the peaks represent nearly similar pattern with different magnitudes.

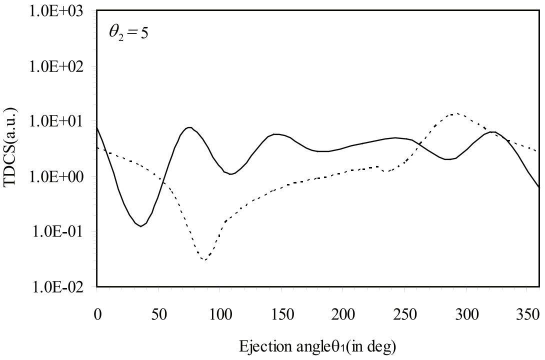

Fig. 1. TDCS for ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV$, $\theta_2 = 5^0$. Solid curve: present first Born result, Dashed curve: hydrogenic 2S-state results [13].

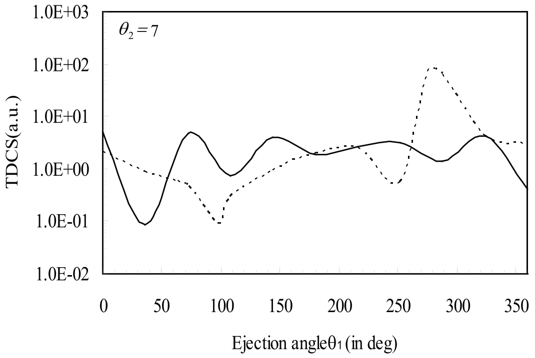

Fig. 2. TDCS for the ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV$, $\theta_2 = 7^0$. Solid curve: present first Born result, Dashed curve: hydrogenic 2S-state results [13].

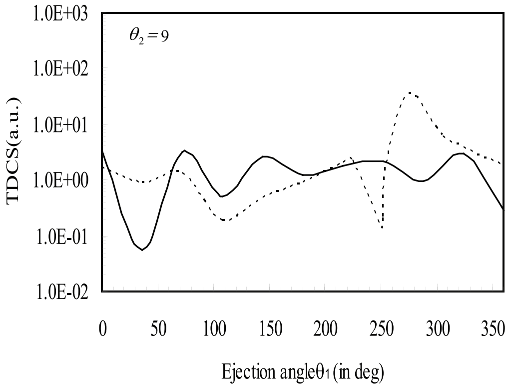

Fig. 3. TDCS for the ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV$, $\theta_2 = 9^0$. Solid curve: present first Born results; Dashed curve: hydrogenic 2S-state results [13].

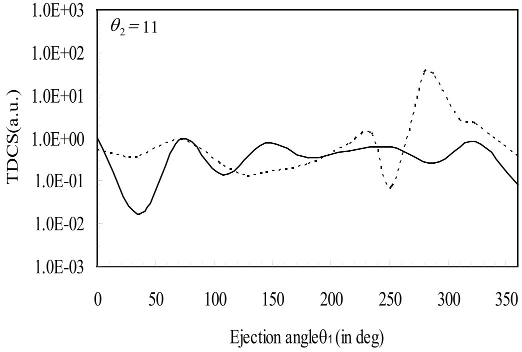

Fig. 4. TDCS for the ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV, \theta_2 = 11^0$. Solid curve: present first Born result ; Dashed curve: hydrogenic 2S-state results [13].

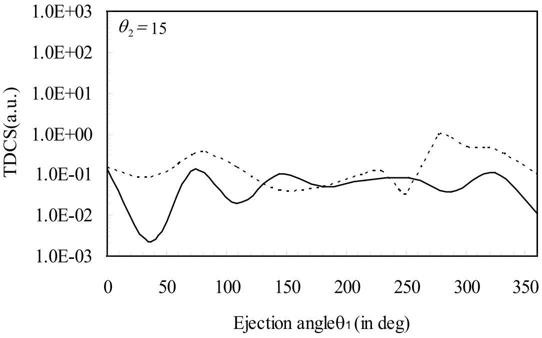

Fig. 5. TDCS for the ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV$, $\theta_2 = 15^0$. Solid curve: present first Born result; Dashed curve: hydrogenic 2S-state results [13].

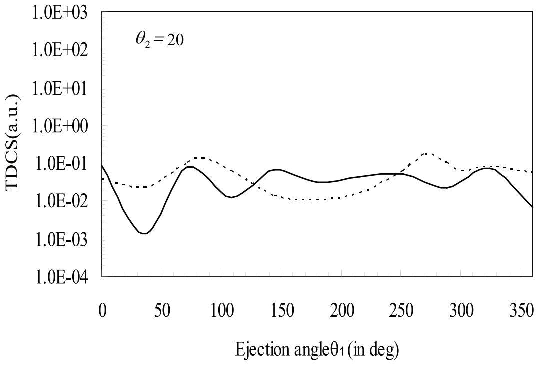

Fig. 6. TDCS for the ionization of hydrogen atoms using electron impact from metastable 2P state varies against the ejected electron angle (θ_1) at $E_i = 250eV$, $E_1 = 5eV$, $\theta_2 = 20^0$. Solid curve: present first Born result, Dashed curve: hydrogenic 2S-state results [13].

It is clear that in some cases, the present first Born TDCS results show qualitative agreement with hydrogenic metastable 2S-state results [13]. But for some other few cases in the binary regions, our results show disagreement in magnitude with compared results [13].

The physical origins of the findings for 250eV incident energy are presented here. The amplitude (the first Born amplitude) corresponds to the first term on the right hand side of equation (6) is defined such that the scattered electrons of momentum $\overline{p}_2$ are described by a plane wave while the ejected electrons of momentum $\overline{p}_1$ are described by a coulomb wave. Also the physical origin of the peaks in the triple differential cross section curves has been investigated, scattering first by atomic nucleus or atomic electron and then by atomic electron for the second time.

4. Conclusions

The present calculation reveals additional possible features of triple differential cross sections (TDCS) curves for small momentum transfer for ionization of hydrogen atoms in metastable 2P- state by 250eV electron impact energy. The present work, applying the multiple scattering theory, provides significant contribution in the field of metastable 2P-state ionization problems. The present study claim experimental set up for better judgment.

5. Acknowledgements

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